1.– Calculate analytically the integral $\int_1^2 \ln(x)dx$. Evaluate numerically the above integral using the Newton-Cotes quadratures of 3 and 4 integration points. Calculate the error involved in each case and compare it with the error bounds of the quadratures. Compare the efficiency of both formulas.

2.– Repeat the above problem for integral $\int_0^1 \ln(x)dx$. Analyze the problems that arise in this case.

3.– Obtain better approximations to the values of the integrals of problems 1 and 2 by combining the results obtained with the 3-point and 4-point quadratures in each case. Study to what extent the hypotheses made in the combination of the quadratures are reliable and analyze the results obtained.

4.– In order to analyze numerically the integral of problem 1 more accurately two alternatives are proposed:

a) Using a Newton-Cotes formula with 7 points.
b) Using the Composite Simpson formula with 7 integration points.

Estimate the error bounds in each case and compare them. Which alternative seems more convenient? Evaluate the integral using both formulas and compare (with each other and with their respective bounds) the errors made.

5.– Write a FORTRAN subroutine that allows to calculate the integral of a function $f(x)$ on an interval $[a, b]$ by means of an adaptive composite Simpson's quadrature. The number of sub-intervals will be doubled at each iteration until the integration error is less than a predetermined value. The integration error will be estimated by Richardson extrapolation..

6.– For a road that is being designed, cross-sectional profiles have been obtained every 5 meters. In each profile, the areas of cut and embankment have been measured with a planimeter and are shown in the attached table. We wish to calculate the volume of earthworks in the cut and fill that would be required in the section between kilometer points 1730 and 1810 of the road. Calculate the volumes by means of the following techniques: **a)** the Trapezoid rule using only the data of the kilometer points that are multiples of 10, **b)** the composite trapezoid rule using all the available data, **c)** Richardson extrapolation between the previously obtained values, and **d)** Simpson's composite rule using all available data. Compare and comment on the results obtained with the different methods.

Kilometer point (m)	Area cut (m^2)	Area fill (m^2)
1730	2.51	0.05
1735	1.32	0.61
1740	1.12	0.82
1745	0.85	0.95
1750	0.63	1.21
1755	0.05	1.35
1760	0.00	1.56
1765	0.00	2.58
1770	0.00	2.41
1775	0.25	2.21
1780	0.56	1.90
1785	0.85	1.50
1790	0.94	0.85
1795	1.57	0.34
1800	1.83	0.11
1805	2.61	0.00
1810	2.57	0.20

TABLA I. Cut and fill areas. Cross-sectional profiles every $5m$.

7.– The following fourth order ODE is to be solved by means of a One-Step Method,

$$u'''' = 3(u')^2 + \frac{9}{2}u^3, \qquad x \in [0, 1],$$

with the boundary conditions

$$u(1) = 4, \qquad u'(1) = 8, \qquad u''(1) = 24, \qquad u'''(1) = 96.$$

a) Propose and develop completely the application of Euler's method and the Improved Euler's method. Explicit how is the order of the ODE reduced and how are the calculations performed in each case.

b) Using both methods, obtain the values of u, u', u'' and u''' at $x = 0.8$ moving from $x = 1$ in two steps.

c) Analyze the results obtained, comparing them and with those of the analytic solution $u(x) = (1 - x/2)^{-2}$.

8.– The following third order ODE is to be solved:

$$u''' = -3uu'', \qquad x \in [-1, 1],$$

with boundary conditions

$$u(-1) = 1, \qquad u'(-1) = -1, \qquad u''(-1) = 2,$$

by means of a One-Step Method.

a) Propose and develop completely the application of Euler's method and the Improved Euler's method. Explicit how is the order of the ODE reduced and how are the calculations performed in each case.

b) Using both methods, obtain the values of u, u' and u'' at $x = 0$ moving from $x = -1$ in two steps.

c) Analyze the results obtained, comparing them and with those of the analytic solution

$$u(x) = (2 + x)^{-1}.$$
