1.– We want to calculate the matrix product $\underset{\sim}{K} = \underset{\sim}{L}\underset{\sim}{U}$ where $\underset{\sim}{L}$ is a lower triangular matrix and $\underset{\sim}{U}$ an upper triangular matrix, both of size n.:

- **a)** What is the shape of matrix $\underset{\sim}{K}$?
- **b)** Design the minimum storage schemes for the three matrices.
- **c)** Write a multiplication algorithm adapted to the above storage schemes.
- **d)** Describe how does the computational cost grow (measured both in terms of the amount of memory and in terms of the computational time required) as a function of the size of the matrices. Compare it with that which would result from storing the complete matrices and using a multiplication algorithm for full matrices.

2.– Repeat the previous problem when the matrices $\underset{\sim}{L}$ and $\underset{\sim}{U}$ have half-bandwidths l and u respectively, with $l << n$, and $u << n$.

3.– Of the matrix $\underset{\sim}{L}$ it is known that it is lower triangular and that all its elements are null except for those located on the main diagonal (which are always nonzero) and those in the row α (which in general will be nonzero, but not necessarily), this is::

$$\underset{\sim}{L} = \begin{bmatrix} l_{11} & & & & & & & \\ & l_{22} & & & & & & \\ & & \ddots & & & & & \\ l_{\alpha 1} & l_{\alpha 2} & \dots & l_{\alpha\alpha} & & & & \\ & & & & \ddots & & & \\ & & & & & l_{\beta\beta} & & \\ & & & & & & \ddots & \\ & & & & & & & l_{nn} \end{bmatrix}.$$

In order to obtain the matrix product $\underset{\sim}{A} = \underset{\sim}{L}\underset{\sim}{L}^T$:

- **a)** Is $\underset{\sim}{A}$ going to be symmetric? And positive definite? Why?
- **b)** What is the general shape of matrix $\underset{\sim}{A}$?
- **c)** Develop the storage schemes that are considered the most suitable for the two matrices. As matrix $\underset{\sim}{A}$ is being computed, is it possible to store its coefficients in the place occupied by the corresponding coefficients of the matrix $\underset{\sim}{L}$ matrix to save memory space? Why?
- **d)** Develop an algorithm suited to perform the matrix product with the storage schemes described in the previous section.
- **e)** How does the computational time necessary to obtain $\underset{\sim}{A}$ grow as the size of the matrix increases?

4.– Of the matrices $\underset{\sim}{L}$ and $\underset{\sim}{U}$ it is known that they are lower and upper triangular respectively, and that all their elements are null except for those located on the main diagonal, in the row α of $\underset{\sim}{L}$ and in the β column of $\underset{\sim}{U}$. This is:

$$
\underset{\sim}{L} = \begin{bmatrix}
l_{11} & & & & & & & \\
& l_{22} & & & & & & \\
& & \ddots & & & & & \\
l_{\alpha 1} & l_{\alpha 2} & \dots & l_{\alpha\alpha} & & & & \\
& & & & \ddots & & & \\
& & & & & l_{\beta\beta} & & \\
& & & & & & \ddots & \\
& & & & & & & l_{nn}
\end{bmatrix}, \quad
\underset{\sim}{U} = \begin{bmatrix}
u_{11} & & & & & u_{1\beta} & & \\
& u_{22} & & & & u_{2\beta} & & \\
& & \ddots & & & \vdots & & \\
& & & u_{\alpha\alpha} & & \vdots & & \\
& & & & \ddots & \vdots & & \\
& & & & & u_{\beta\beta} & & \\
& & & & & & \ddots & \\
& & & & & & & u_{nn}
\end{bmatrix}
$$

a) what is the general shape of the product matrix $\underset{\sim}{A} = \underset{\sim}{L}\underset{\sim}{U}$?
b) Design storage schemes suited for the three matrices
c) Write a specific algorithm including the storage schemes described for the matrix product.