NUMERICAL METHODS AND PROGRAMMING **Year 2024–20245**

Storage of numbers and algorithms **(PROBLEMS 1)**

1.– Write in fixed point and floating point, and in the denary, binary, octal and hexadecimal number bases the following numbers:

a) $5,3125$
b) $0,1$
c) $1/3$
d) π
e) e
f) 52745916

2.– The above values are to be store in the variables `A`, `UD`, `UT`, `PI`, `E` y `GR` of a FORTRAN program:

```
REAL*4 A,UD,UT,PI,E,GR
A=5.3125
UD=0.1
UT=0.333333
PI=3.14159
E=2.71828
GR=52745916.
...
```

If for the mantissa of the numbers `REAL*4` 3 bytes are used (sign included):

a) Obtain the values that are actually stored in each case.
b) Compare the stored values with the corresponding exact values, indicating the sources of error.
c) Compare the rounding error produced in each case with its upper bound
d) Estimate how many significant figures should have been used to write the numbers. $1/3$, π and e. Repeat the estimation for the case in which the variables are to be stored as `REAL*8`.

3.– Compare the values stored in the variables `GDIF` and `NGDIF` in the following FORTRAN program:

```
REAL*4 GDIF
INTEGER*4 NGDIF
GDIF=52745916.+15.
GDIF=GDIF-52745916.
...
NGDIF=52745916+15
NGDIF=NGDIF-52745916
...
IF (GDIF.EQ.FLOAT(NGDIF)) STOP
...
```

If for the mantissa of the variables `REAL*4` 3 bytes are used (sign included), will the program stop when executing the instruction `IF`? What conclusions can be drawn from this example?

4.– A given computer takes a tenth of a second (at most) to:

- **a)** Obtain, through the equations of minimum least squares, the regression line that best approximates 100 data points.
- **b)** Calculate the modulus of a vector of 500 elements.
- **c)** Sort from highest to lowest a list of 50 numbers by the Bubble Sort method.
- **d)** Obtain the product of a square matrix of size 20 times a vector.
- **e)** Multiply to square matrices of size 8.
- **f)** Determine (by algebraic definition) the determinant of a square matrix of size 5.
- **g)** Find by sequential search a telephone number in a list (alphabetically sorted) of 100 subscribers.
- **h)** Find by bisection a telephone number in a list (alphabetically sorted) of 100 subscribers.

What is going to be the computational time if the size of the problem (size of the matrix, number of date, for each case) is ten, a hundred, a thousand, ten thousand, a hundred thousand or a million times larger?

5.– In order to obtain numerically the value of $\alpha = \sqrt{2}$ two iterative algorithms are proposed:

$$\textbf{1)}\ x_{n+1} = \frac{2 + x_n(10 - x_n)}{10}; \qquad \textbf{2)}\ x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n}.$$

For both of them:

- **a)** Analyze the evolution of the absolute truncation algorithm between two consecutive iterations.
- **b)** Determine for which initial values x_0 does the algorithm converges and for which ones it does not. Provide examples.
- **c)** Simplify the above study assuming a "sufficiently good" initial approximation x_0. Determine the order of convergence.
- **d)** Knowing that the value $\sqrt{2}$ is in between 1.41 and 1.42, estimate how many iterations are needed starting at an initial approximation of $x_0 = 1.415$ in order to obtain a better approximation with 5, 10, 15, 20 y 100 exact significant digits. Perform several iterations to verify the resutls.

6.– An engineer working on road layout is creating a FORTRAN program. One part of the program has to print a list of the cut and fill areas along the road axis. By default the listing starts at kilometer point (PK) 0.000 and a data is printed every 0.1 km. The program works apparently well, but users complain that sometimes strange values appear in the PK column, such as 70.999 instead of 71.000.

Se pide:

- **a)** Make a FORTRAN program that repeatedly adds the value 0.1 in a variable of type `REAL*4` (initialized to zero) and prints the results. Verify that after a certain number of operations the printed result is not a multiple of $0,1$. Repeat the calculations with variables of type `REAL*8`, and check that the effect persists, although it takes longer to occur.
- **b)** Check that the same effect occurs in a spreadsheet. To do this, start a column with the value zero in the first row and generate the values of each of the following rows by adding the value 0.1 to the result of the previous row.
- **c)** Propose an alternative to solve the problem satisfactorily.
