1.a) $$I = \int_{y=0}^{y=1} \left[\underbrace{\int_{x=1}^{x=2} 1 \, dx}_{g(y) = 1} \right] dy = 1$$

1.b) $$I = \int_{y=1}^{y=2} \left[\underbrace{\int_{x=0}^{x=3} (x+y) \, dx}_{g(y) = 9/2 + 3y} \right] dy = 9$$

1.c) $$I = \int_{x=2}^{x=4} \left[\underbrace{\int_{y=1}^{y=2} (x^2+y^2) \, dy}_{g(x) = x^2 + 7/3} \right] dx = 70/3$$

1.d) $$I = \int_{x=0}^{x=1} \left[\underbrace{\int_{y=x^2}^{y=x} (xy^2) \, dy}_{g(x) = x^4/3 - x^7/3} \right] dx = 1/40$$

1.e) $$I = \int_{y=1}^{y=2} \left[\underbrace{\int_{x=0}^{x=y^{3/2}} (x/y^2) \, dx}_{g(y) = y/2} \right] dy = 3/4$$

1.f) $$I = \int_{x=0}^{x=1} \left[\underbrace{\int_{y=x}^{y=\sqrt{x}} (y+y^3) \, dy}_{g(x) = x/2 - x^2/4 - x^4/4} \right] dx = 7/60$$

1.g) $$I = \int_{x=0}^{x=1} \left[\underbrace{\int_{y=0}^{y=x^2} (x e^y) \, dy}_{g(x) = x(e^{(x^2)} - 1)} \right] dx = e/2 - 1$$

1.h) $$I = \int_{y=2}^{y=4} \left[\underbrace{\int_{x=y}^{x=8-y} y \, dx}_{g(y) = 8y - 2y^2} \right] dy = 32/3$$

1.i) $$I = \int_{\theta=0}^{\theta=\operatorname{arctg}(3/2)} \left[\underbrace{\int_{\rho=0}^{\rho=2\sec(\theta)} \rho \, d\rho}_{g(\theta) = 2/\cos^2\theta} \right] d\theta = 3$$

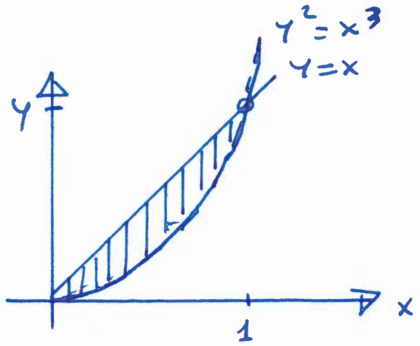
1.j) $$I = \int_{\theta=0}^{\theta=\pi/2} \left[\underbrace{\int_{\rho=0}^{\rho=2} (\rho^2 \cos\theta) \, d\rho}_{g(\theta) = 8/3 \cos\theta} \right] d\theta = 8/3$$

1.k) $$I = \int_{\theta=0}^{\theta=\pi/4} \left[\underbrace{\int_{\rho=0}^{\rho=\operatorname{tg}\theta \cdot \sec(\theta)} (\rho^3 \cos^2\theta) \, d\rho}_{g(\theta) = 1/4 \operatorname{tg}^4\theta / \cos^2\theta} \right] d\theta = 1/20$$

1.l) $$I = \int_{\theta=0}^{\theta=2\pi} \left[\underbrace{\int_{\rho=0}^{\rho=1-\cos\theta} (\rho^3 \cos^2\theta) \, d\rho}_{g(\theta) = \cos^2\theta (1-\cos\theta)^4/4} \right] d\theta = 49\pi/32 \quad (*)$$

(*) Se utiliza $I_n = \int_0^{2\pi} (\cos\theta)^n \, d\theta \Longrightarrow I_0 = 2\pi$, $I_1 = 0$, $I_n = \frac{n-1}{n} I_{n-2}$ para $n \geq 2$

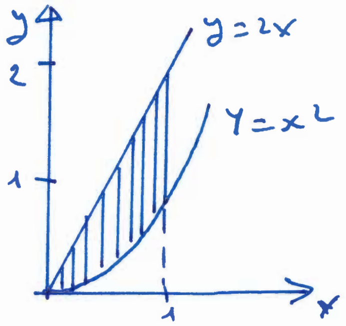
2.a)



$$I = \int_{x=0}^{x=1} \Bigg[\underbrace{\int_{y=x^{3/2}}^{y=x} 1 \, dy}_{g(x)} \Bigg] dx$$

$g(x) = x - x^{3/2}$; $I = 1/10$

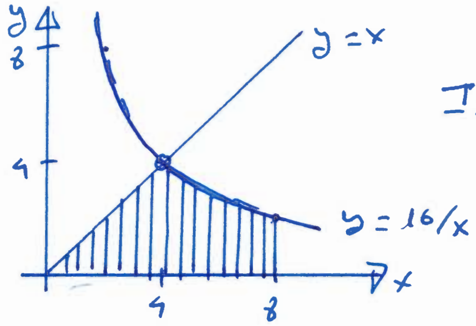
2.b)



$$I = \int_{x=0}^{x=1} \Bigg[\underbrace{\int_{y=x^2}^{y=2x} 1 \, dy}_{g(x)} \Bigg] dx$$

$g(x) = 2x - x^2$; $I = 2/3$

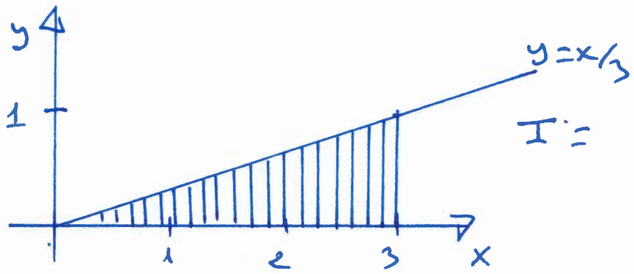
2.c)



$$I = \int_{x=0}^{x=4} \Bigg[\underbrace{\int_{y=0}^{y=x} x^2 dy}_{g_1(x)} \Bigg] dx + \int_{x=4}^{x=8} \Bigg[\underbrace{\int_{y=0}^{y=16/x} x^2 dy}_{g_2(x)} \Bigg] dx$$

$g_1(x) = x^3$; $g_2(x) = 16x$; $I = 448$

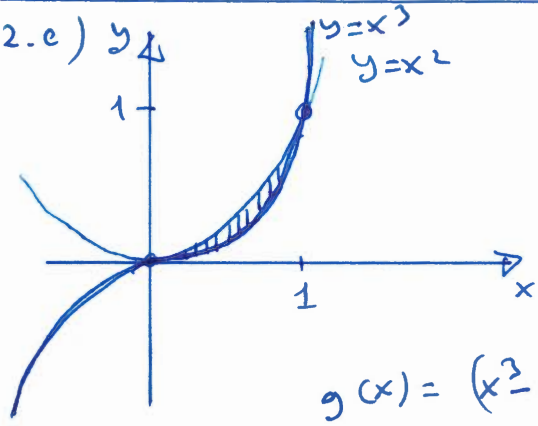
2.d)



$$I = \int_{x=0}^{x=3} \Bigg[\underbrace{\int_{y=0}^{y=x/3} e^{x^2} dy}_{g(x)} \Bigg] dx$$

$g(x) = e^{x^2} \cdot x/3$; $I = 1/6\,(e^9 - 1)$

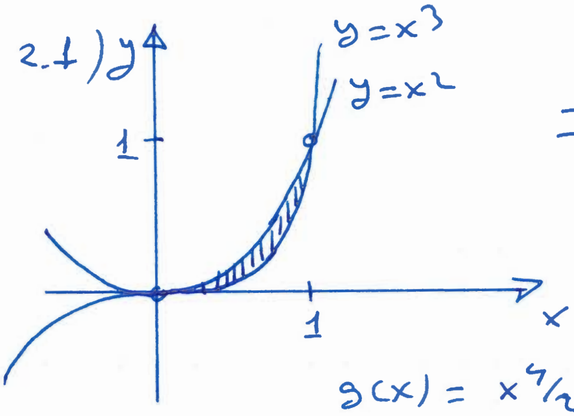
2.e)



$$I = \int_{x=0}^{x=1} \Bigg[\underbrace{\int_{y=x^3}^{y=x^2} x \, dy}_{g(x)} \Bigg] dx$$

$g(x) = (x^3 - x^4)$; $I = 1/20$

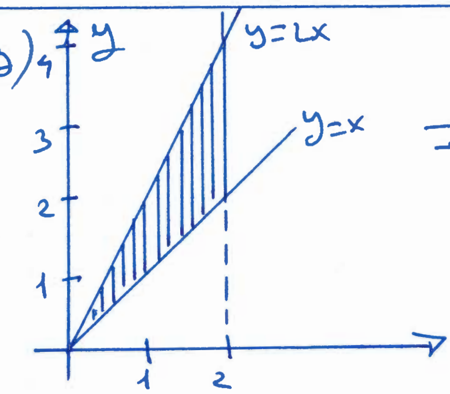
2.f)



$$I = \int_{x=0}^{x=1} \underbrace{\left[\int_{y=x^3}^{y=x^2} y\, dy \right]}_{g(x)} dx$$

$g(x) = x^4/2 - x^6/2$; $I = 1/35$

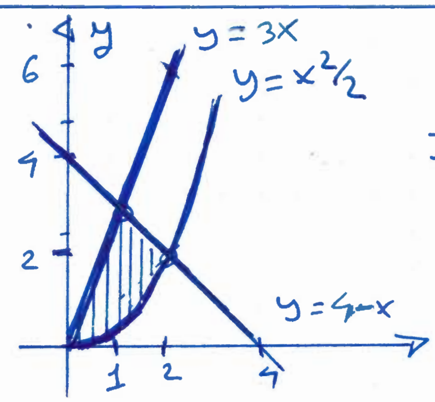
2.g)



$$I = \int_{x=0}^{x=2} \underbrace{\left[\int_{y=x}^{y=2x} x^2\, dy \right]}_{g(x)} dx$$

$g(x) = x^3$; $I = 4$

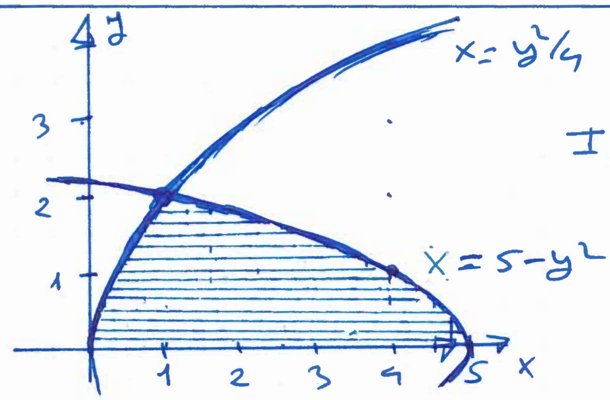
2.h)



$$I = \int_{x=0}^{x=1} \underbrace{\left[\int_{y=x^2/2}^{y=3x} dy \right]}_{g_1(x)} dx + \int_{x=1}^{x=2} \underbrace{\left[\int_{y=x^2/2}^{y=4-x} dy \right]}_{g_2(x)} dx$$

$g_1(x) = 3x - x^2/2$; $g_2(x) = 4 - x - x^2/2$; $I = 8/3$

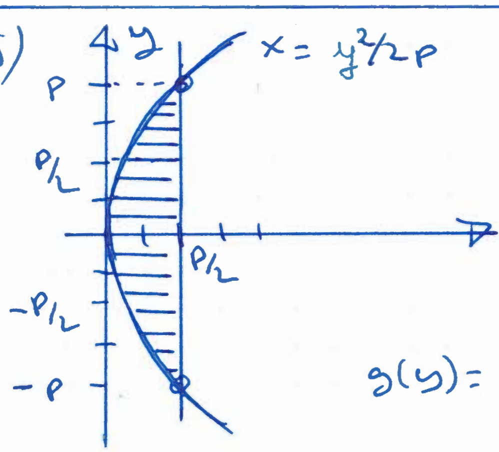
2.i)



$$I = \int_{y=0}^{y=2} \underbrace{\left[\int_{x=y^2/4}^{x=5-y^2} y\, dx \right]}_{g(y)} dy$$

$g(y) = 5y - 5y^3/4$; $I = 5$

2.j)



$$I = \int_{y=-p}^{y=p} \underbrace{\left[\int_{x=y^2/2p}^{x=p/2} x y^2\, dx \right]}_{g(y)} dy$$

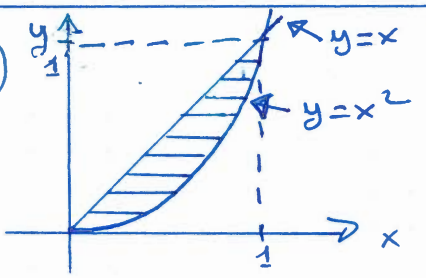
$g(y) = y^2 p^2/8 - y^6/(8p^2)$; $I = p^5/21$

3.a) $$I = \int_{x=1}^{x=2} \underbrace{\left[\int_{y=0}^{y=1} 1 \, dy \right]}_{g(x) = 1} dx = 1$$

3.b) $$I = \int_{x=0}^{x=3} \underbrace{\left[\int_{y=1}^{y=2} (x+y) \, dy \right]}_{g(x) = x + 3/2} dx = 9$$

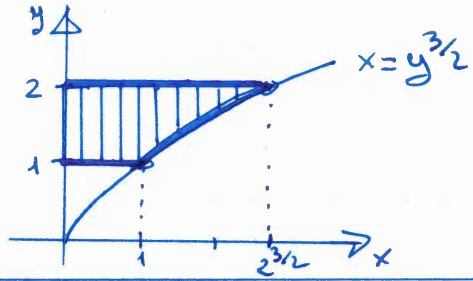
3.c) $$I = \int_{y=1}^{y=2} \underbrace{\left[\int_{x=2}^{x=4} (x^2+y^2) \, dx \right]}_{g(y) = 56/3 + 2y^2} dy = 70/3$$

3.d)



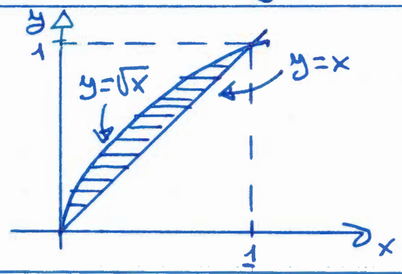
$$I = \int_{y=0}^{y=1} \underbrace{\left[\int_{x=y}^{x=\sqrt{y}} (xy^2) \, dx \right]}_{g(y) = y^3/2 - y^4/2} dy = 1/40$$

3.e)



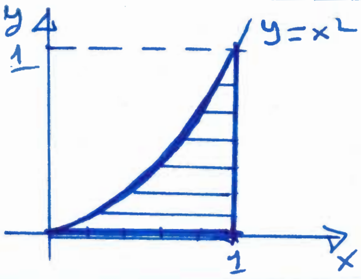
$$I = \int_{x=0}^{x=1} \underbrace{\left[\int_{y=1}^{y=2} (x/y^2) \, dy \right]}_{g_1(x) = x/2} dx + \int_{x=1}^{x=2^{3/2}} \underbrace{\left[\int_{y=x^{2/3}}^{y=2} (x/y^2) \, dy \right]}_{g_2(x) = x^{1/3} - x/2} dx = 3/4$$

3.f)



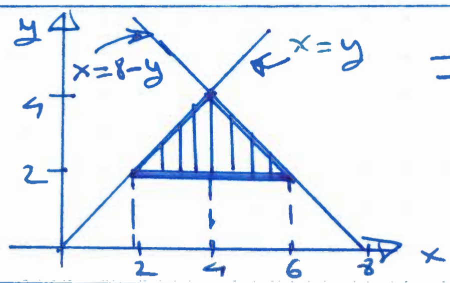
$$I = \int_{y=0}^{y=1} \underbrace{\left[\int_{x=y^2}^{x=y} (y+y^3) \, dx \right]}_{g(y) = y^2 - y^3 + y^4 - y^5} dy = 7/60$$

3.g)



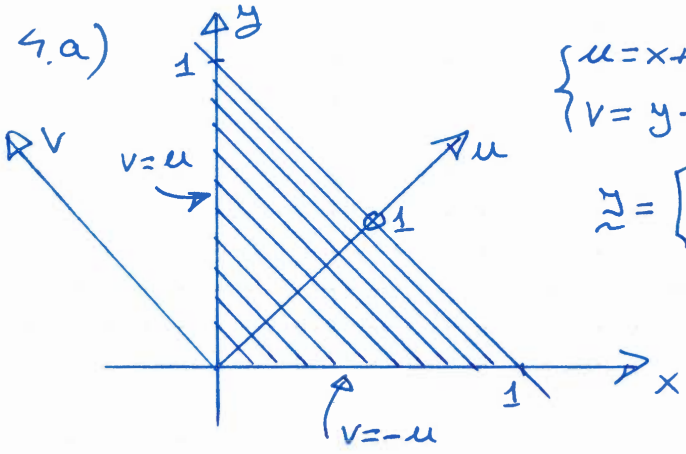
$$I = \int_{y=0}^{y=1} \underbrace{\left[\int_{x=\sqrt{y}}^{x=1} (x e^y) \, dx \right]}_{g(y) = 1/2 (1-y) e^y} dy = e/2 - 1$$

3.h)



$$I = \int_{x=2}^{x=4} \underbrace{\left[\int_{y=2}^{y=x} y \, dy \right]}_{g_1(x) = x^2/2 - 2} dx + \int_{x=4}^{x=6} \underbrace{\left\{ \int_{y=2}^{y=8-x} y \, dy \right]}_{g_2(x) = (8-x)^2/2 - 2} dx = 32/3$$

4.a)

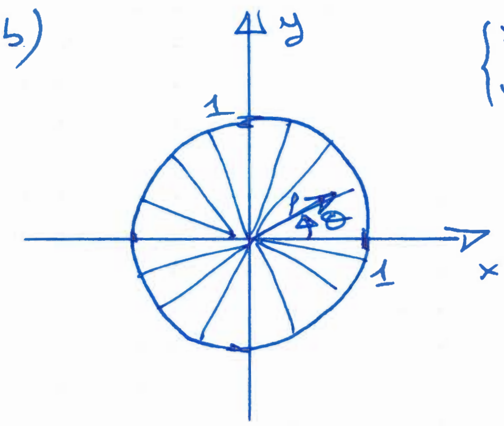


$$\begin{cases} u = x+y \\ v = y-x \end{cases} \to \begin{cases} x = (u-v)/2 \\ y = (u+v)/2 \end{cases}$$

$$\underset{\sim}{J} = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 \\ 1/2 & 1/2 \end{bmatrix} \Rightarrow J = 1/2$$

$$I = \int_{u=0}^{u=1} \Big[\underbrace{\int_{v=-u}^{v=u} \left(e^{v/u}\right) \cdot \tfrac{1}{2} \cdot dv}_{g(u) = \frac{1}{2}(e - 1/e)u} \Big] du = \frac{1}{4}\left(e - \frac{1}{e}\right)$$

4.b)

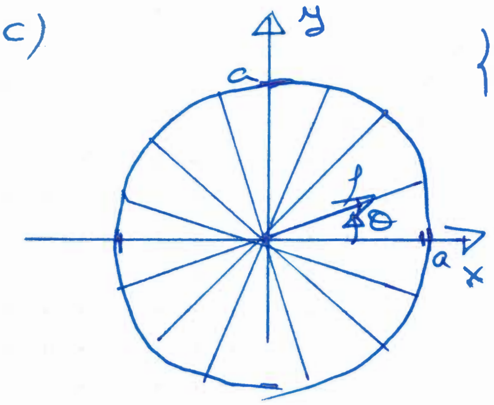


$$\begin{cases} x = \rho \cos\theta \\ y = \rho \operatorname{sen}\theta \end{cases}$$ (POLARES)

$$\underset{\sim}{J} = \begin{bmatrix} \partial x/\partial\rho & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\rho \operatorname{sen}\theta \\ \operatorname{sen}\theta & \rho\cos\theta \end{bmatrix} \Rightarrow J = \rho$$

$$I = \int_{\theta=0}^{\theta=2\pi} \Big[\underbrace{\int_{\rho=0}^{\rho=1} \left(\sqrt{1-\rho^2}\right) \cdot \rho \cdot d\rho}_{g(\theta) = 1/3} \Big] d\theta = 2\pi/3$$

4.c)

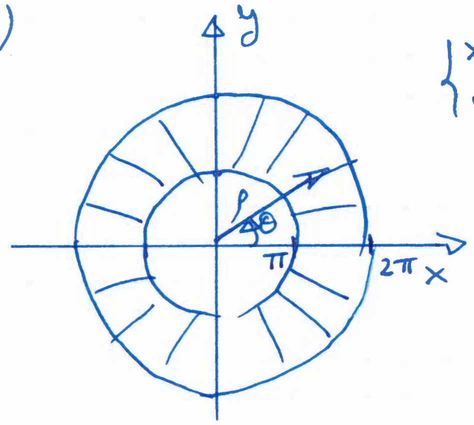


$$\begin{cases} x = \rho \cos\theta \\ y = \rho \operatorname{sen}\theta \end{cases}$$ (POLARES)

$$\underset{\sim}{J} = \cdots \qquad \Rightarrow J = \rho$$

$$I = \int_{\theta=0}^{\theta=2\pi} \Big[\underbrace{\int_{\rho=0}^{\rho=a} \rho \cdot \rho \cdot d\rho}_{g(\theta) = a^3/3} \Big] d\theta = 2\pi/3 \, a^3$$

4.d)

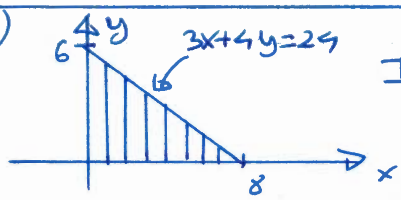


$$\begin{cases} x = \rho \cos\theta \\ y = \rho \operatorname{sen}\theta \end{cases}$$ (POLARES)

$$\underset{\sim}{J} = \cdots \qquad \Rightarrow J = \rho$$

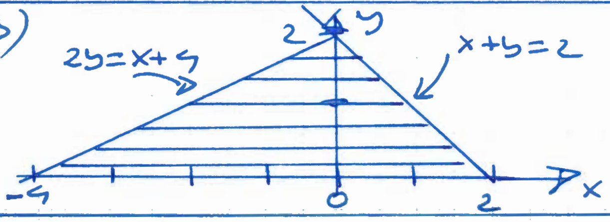
$$I = \int_{\theta=0}^{\theta=2\pi} \Big[\underbrace{\int_{\rho=\pi}^{\rho=2\pi} (\operatorname{sen}\rho) \cdot \rho \, d\rho}_{g(\theta) = -3\pi} \Big] d\theta = -6\pi^2$$

5.a)



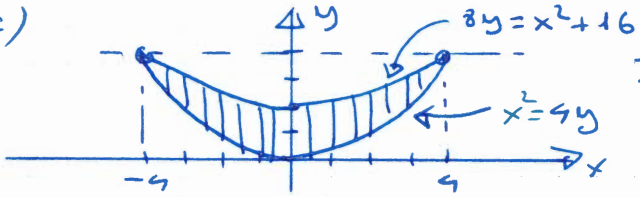
$$I=\int_{x=0}^{x=8}\left[\underbrace{\int_{y=0}^{y=\frac{24-3x}{4}} 1\cdot dy}_{g(x)=6-3/4\,x}\right]dx = 24$$

5.b)



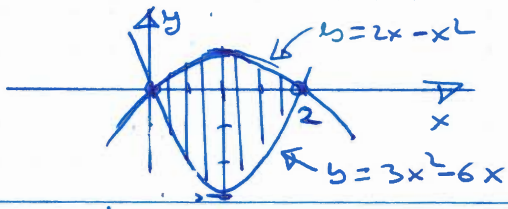
$$I=\int_{y=0}^{y=2}\left[\underbrace{\int_{x=2y-4}^{x=2-y} 1\,dx}_{g(y)=6-3y}\right]dy = 6$$

5.c)



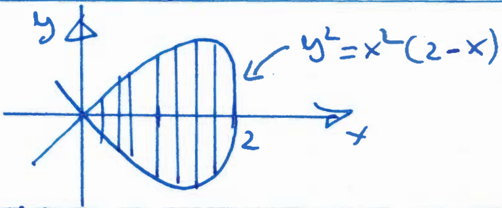
$$I=\int_{x=-4}^{x=4}\left[\underbrace{\int_{y=x^2/4}^{y=x^2/8+2} 1\cdot dy}_{g(x)=2-x^2/8}\right]dx = 32/3$$

5.d)



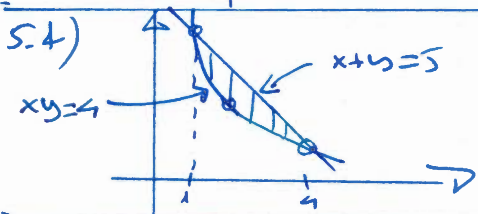
$$I=\int_{x=0}^{x=2}\left[\underbrace{\int_{y=3x^2-6x}^{y=2x-x^2} 1\,dy}_{g(x)=8x-4x^2}\right]dx = 16/3$$

5.e)



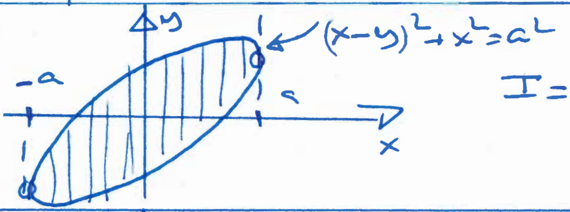
$$I=\int_{x=0}^{x=2}\left[\underbrace{\int_{y=-x\sqrt{2-x}}^{y=+x\sqrt{2-x}} 1\,dy}_{g(x)=2x\sqrt{2-x}}\right]dx = \frac{32\sqrt{2}}{15}$$

5.f)



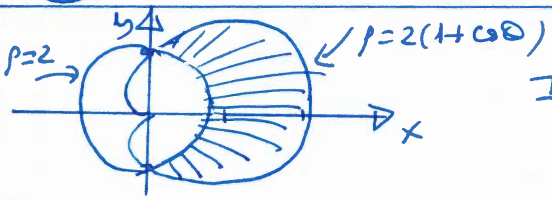
$$I=\int_{x=1}^{x=4}\left[\underbrace{\int_{y=4/x}^{y=5-x} 1\,dy}_{g(x)=5-x-4/x}\right]dx = 15/2-8\ln(2)$$

5.g)



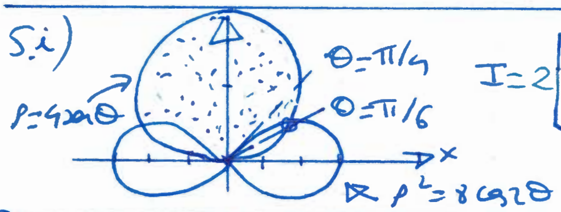
$$I=\int_{x=-a}^{x=a}\left[\underbrace{\int_{y=x-\sqrt{a^2-x^2}}^{y=x+\sqrt{a^2-x^2}} 1\,dy}_{g(x)=2\sqrt{a^2-x^2}}\right]dx = \pi a^2$$

5.h)



$$I=\int_{\theta=-\pi/2}^{\theta=\pi/2}\left[\underbrace{\int_{\rho=2}^{\rho=2(1+\cos\theta)} 1\ \rho\,d\rho}_{g(\theta)=2\left((1+\cos\theta)^2-1\right)}\right]d\theta = 8+\pi$$

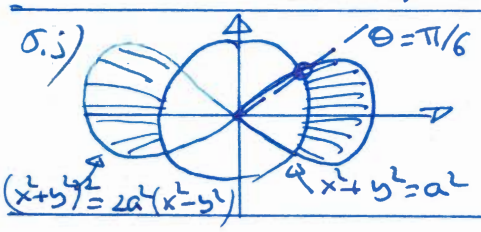
5.i)



$$I=2\left[\int_{\theta=\pi/6}^{\theta=\pi/2}\left[\underbrace{\int_{\rho=0}^{\rho=4\sin\theta} 1\cdot\rho\,d\rho}_{g_1(\theta)=8\sin^2\theta}\right]d\theta-\int_{\theta=\pi/6}^{\theta=\pi/4}\left[\underbrace{\int_{\rho=0}^{\rho=\sqrt{8\cos(2\theta)}} 1\cdot d\rho}_{g_2(\theta)=4\cos2\theta}\right]d\theta\right]$$

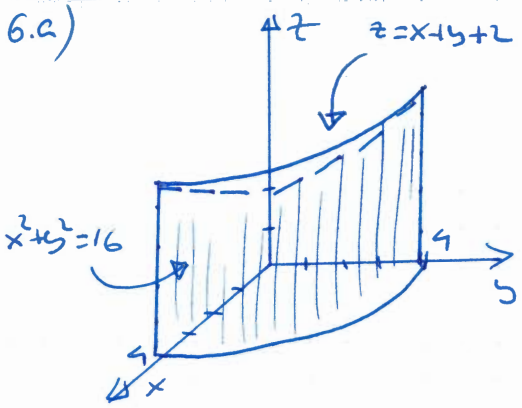
$$I=8\pi/3-4+4\sqrt{3}$$

5.j)



$$I=4\left[\int_{\theta=0}^{\theta=\pi/6}\left[\underbrace{\int_{\rho=a}^{\rho=\sqrt{2a^2\cos2\theta}} 1\cdot\rho\,d\rho}_{g(\theta)=a^2(\cos2\theta-1/2)}\right]d\theta\right]=\frac{a^2}{3}\left(3\sqrt{3}-\pi\right)$$

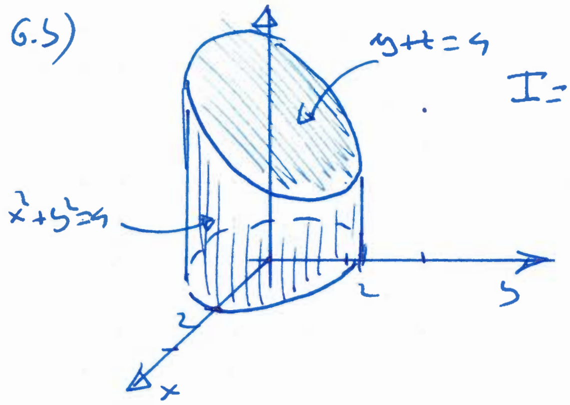
6.a)



$$I = \int_{\theta=0}^{\theta=\pi/2} \Bigg[\underbrace{\int_{\rho=0}^{\rho=4} (\rho\cos\theta + \rho \operatorname{sen}\theta + 2)\,\rho\, d\rho}_{g(\theta) = 64/3\,(\cos\theta + \operatorname{sen}\theta) + 16} \Bigg] d\theta$$

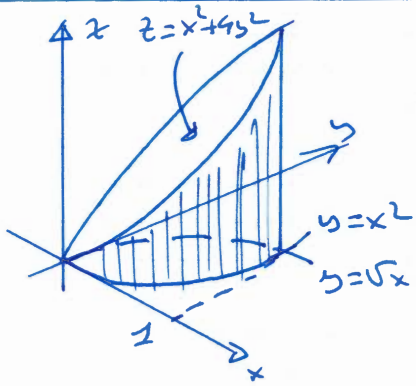
$$\Rightarrow I = 128/3 + 8\pi$$

6.b)



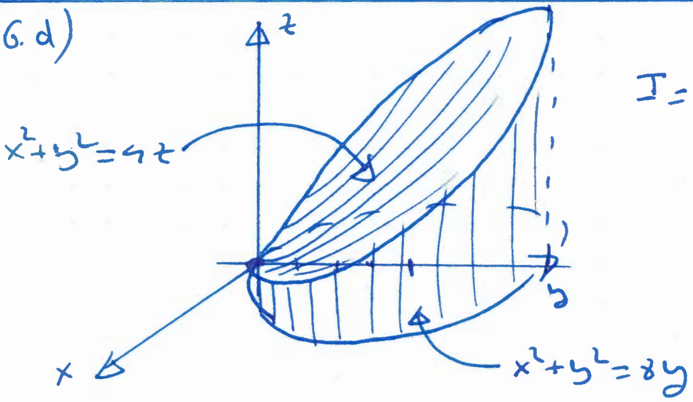
$$I = \int_{\theta=0}^{\theta=2\pi} \Bigg[\underbrace{\int_{\rho=0}^{\rho=2} (4 - \rho \operatorname{sen}\theta)\,\rho\, d\rho}_{g(\theta) = 8(1 - \operatorname{sen}\theta/3)} \Bigg] d\theta = 16\pi$$

6.c)



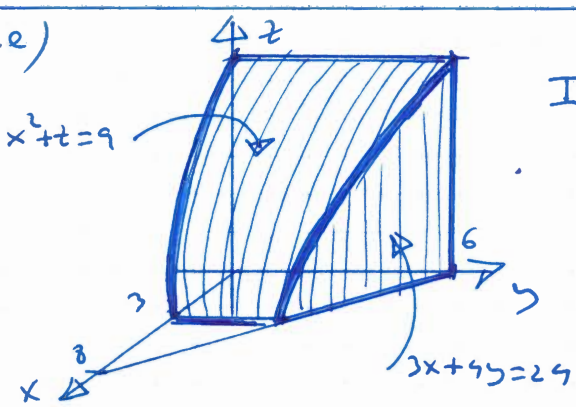
$$I = \int_{x=0}^{x=1} \Bigg[\underbrace{\int_{y=x^2}^{y=\sqrt{x}} (x^2 + 4y^2)\, dy}_{g(x) = 4/3\, x^{3/2} + x^{5/2} - x^4 - 4/3\, x^6} \Bigg] dx = 3/7$$

6.d)



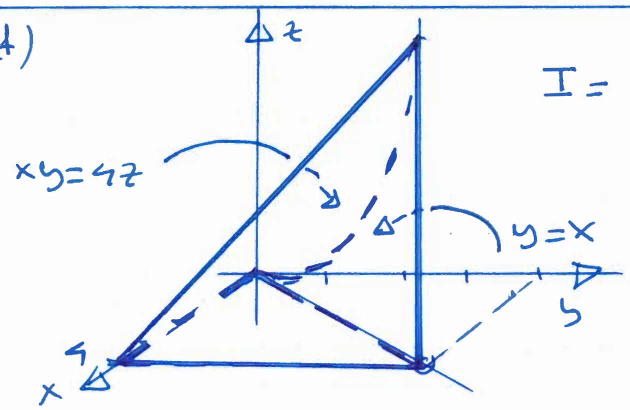
$$I = \int_{\theta=0}^{\theta=\pi} \Bigg[\underbrace{\int_{\rho=0}^{\rho=8\operatorname{sen}\theta} (\rho^2/4)\,\rho\, d\rho}_{g(\theta) = 256 \operatorname{sen}^4\theta} \Bigg] d\theta = 96\pi$$

6.e)



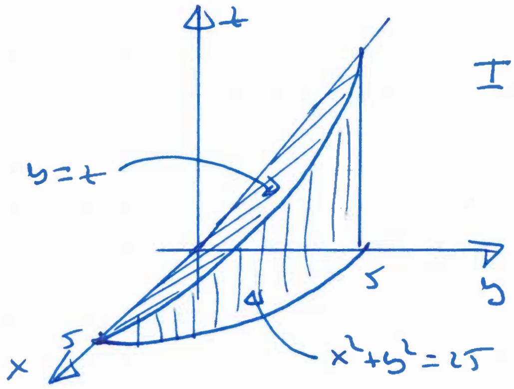
$$I = \int_{x=0}^{x=3} \Bigg[\underbrace{\int_{y=0}^{y=\frac{24-3x}{4}} (9 - x^2)\, dy}_{g(x) = (9 - x^2)(6 - 3/4\, x)} \Bigg] dx = 1485/16$$

6.f)



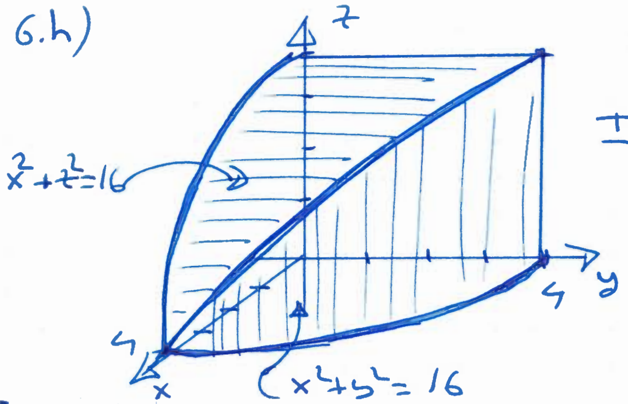
$$I = \int_{x=0}^{x=4} \Bigg[\underbrace{\int_{y=0}^{y=x} \left(\frac{xy}{4}\right) dy}_{g(x) = x^3/8} \Bigg] dx = 8$$

6.g)



$$I = \int_{\theta=0}^{\theta=\pi/2} \Bigg[\underbrace{\int_{\rho=0}^{\rho=5} (\rho \operatorname{sen}\theta)\, \rho\, d\rho}_{g(\theta) = \frac{125}{3} \operatorname{sen}\theta} \Bigg] d\theta = 125/3$$

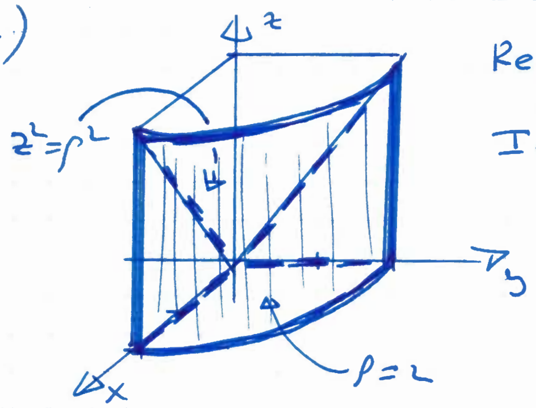
6.h)



Representamos un octante.

$$I = 8 \int_{x=0}^{x=4} \Bigg[\underbrace{\int_{y=0}^{y=\sqrt{16-x^2}} \sqrt{16-x^2}\, dy}_{g(x) = (16-x^2)} \Bigg] dx = \frac{1024}{3}$$

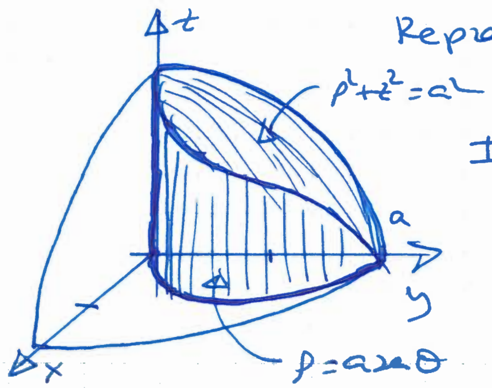
6.i)



Representamos un octante

$$I = 8 \int_{\theta=0}^{\theta=\pi/2} \Bigg[\underbrace{\int_{\rho=0}^{\rho=2} \rho \cdot \rho\, d\rho}_{g(\theta) = 8/3} \Bigg] d\theta = \frac{32}{3}\pi$$

6.j)

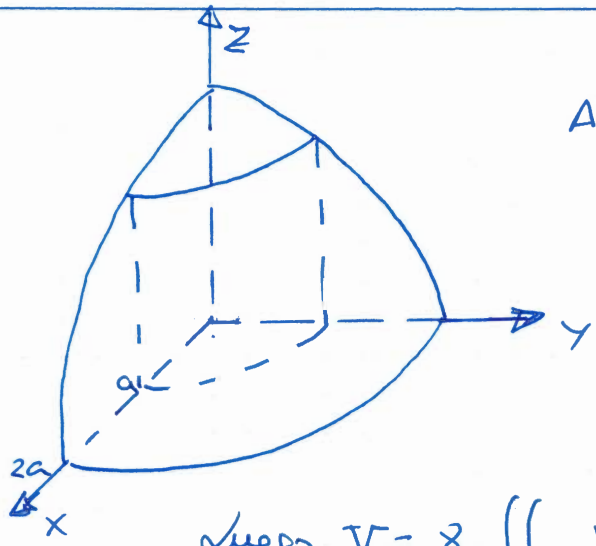


Representamos un octante

$$I = 4 \int_{\theta=0}^{\theta=\pi/2} \Bigg[\underbrace{\int_{\rho=0}^{\rho=a\operatorname{sen}\theta} \sqrt{a^2-\rho^2}\, \rho\, d\rho}_{g(\theta) = 1/3\, a^3 (1-\cos^3\theta)} \Bigg] d\theta$$

$$I = 2a^3/9\, (3\pi - 4)$$

7)



Analizamos un octante:

$$\begin{cases} \text{Esfera: } x^2+y^2+z^2=(2a)^2 \\ \text{Cilindro: } x^2+y^2=a^2 \end{cases}$$

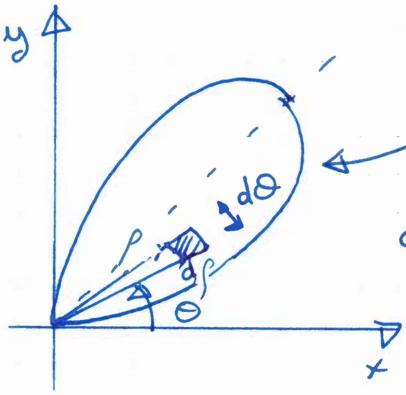
Luego $V = 8 \iint_{C_1} \sqrt{(2a)^2-x^2-y^2}\,dx\,dy$

siendo C_1 la región $x^2+y^2 \leq a^2$, $x \geq 0$, $y \geq 0$

En polares: $\begin{cases} x=\rho\cos\theta \\ y=\rho\,\mathrm{sen}\,\theta \end{cases} \Rightarrow \underset{\sim}{J} = \begin{bmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\rho\,\mathrm{sen}\,\theta \\ \mathrm{sen}\,\theta & \rho\cos\theta \end{bmatrix} \Rightarrow J=\rho$

$$V = 8\int_{\theta=0}^{\theta=\pi/2} \Big[\underbrace{\int_{\rho=0}^{\rho=a} \left(\sqrt{(2a)^2-\rho^2}\right)\rho\,d\rho}_{g(\theta)=\frac{a^3}{3}(8-3\sqrt{3})} \Big] d\theta = \frac{4}{3}\pi a^3 (8-3\sqrt{3})$$

8)



$\rho = \mathrm{sen}(2\theta) \quad \theta \in [0, \pi/2]$

$dV = 2\pi\,(\rho\,\mathrm{sen}\,\theta)\,\rho\,d\rho\,d\theta$

$$V = \int_{\theta=0}^{\theta=\pi/2} \Big[\underbrace{\int_{\rho=0}^{\rho=\mathrm{sen}(2\theta)} \left(2\pi\rho^2\,\mathrm{sen}\,\theta\right) d\rho}_{g(\theta)=\frac{2\pi}{3}\mathrm{sen}\,\theta\,(\mathrm{sen}(2\theta))^3} \Big] d\theta = 32\pi/105$$