LINEAR ALGEBRA II — Problem Set 4.1

Conics (Academic Year 2022–2023)

NOTE: All problems posed in the Euclidean affine plane endowed with a rectangular reference system.

1.– Consider a conic given by the equation:

$$x^2 - 4xy + y^2 - 6x + 2y = 0$$

Calculate the equation of the tangent lines to the conic that pass through the point $(-1, -3)$.

2.– In the affine plane given the conic of equation:

$$x^2 - 2xy + y^2 + 4x + 1 = 0$$

(i) Classify the conic.

(ii) Find its center, axes, vertices, and asymptotes.

(iii) Calculate its reduced equation, eccentricity and distance from the vertex to the focus.

(Final exam, May 2018)

3.– We consider the conic given by the equation

$$3y^2 - 4xy + 12x - 14y + 19 = 0$$

b) Find its asymptotes.

c) Obtain the exterior tangents to the conic passing through the point $(0, 3)$.

4.– For the following conics

(1) $3x^2 + 3y^2 + 2xy - 4x - 4y = 0$

(2) $2x^2 + 3y^2 - 4x + 6y + 6 = 0$

(3) $6y^2 + 8xy - 8x + 4y - 8 = 0$

(4) $x^2 + 4y^2 - 4xy + 4 = 0$

(5) $x^2 - 2y^2 + xy + xy = 0$

(6) $x^2 + y^2 + 4x + 4 = 0$

(7) $x^2 + y^2 + 2xy - xy - 2 = 0$

(8) $x^2 + 4y^2 - 4xy + 6y = 0$

(9) $4x^2 + 4y^2 + 8xy + 4x + 4y + 1 = 0,$

determine: centers, singular points, asymptotic directions, asymptotes, axes, vertices.

5.– For the conics of the previous problem, it is requested:

(a) Classify them without finding the reduced equations.

(b) Give the reduced equations of the nondegenerate ones and the lines that form the degenerate ones.

(c) In the cases where they exist, determine foci, directrices and eccentricity.

6.– Given the curve of equation

$$3y^2 + 4xy - 4x - 6y - 1 = 0.$$

(i) Classify this conic and give its reduced equation.

(ii) Find the point whose polar line is $x - 1 = 0$.

(iii) Find its foci.

(Final exam, May 2013)

7.– In the affine plane the family of conics is considered:

$$x^2 + 2axy + 2y^2 + 2x - 6ay + 1 = 0, \qquad a \in \mathbb{R}.$$

(i) Classify the conics in terms on the parameter a.

(ii) For $a = 1$ calculate the center of the conic.

(iii) For $a = \sqrt{2}$ calculate the distance between the vertex and the focus.

(iv) For which values of a is the eccentricity of the conic greater than 1?

(Final exam, May 2016)

8.– In the affine plane we consider the conic with equation

$$x^2 + 2xy + y^2 - 4x - 1 = 0$$

(i) Classify the conic and find its reduced equation. (0.6 points)

(ii) Find its eccentricity, asymptotes, and the distance between a focus and the vertex which is closest to it. (0.4 points)

(iii) Find the tangent lines to the conic that pass through the point $(-1, 2)$. (0.5 points)

(iv) Find the equation of a conic that has the same tangent lines as the given curve at the points $(0, -1)$ and $(2, 1)$ and passes through the origin. (1 point)

(Final exam, June 2020)

9.– In the affine plane we consider the conic with the equation

$$x^2 + 2kxy + y^2 + 2ky = 0$$

(i) Classify the conic in terms of the values of k

(ii) For $k = 2$ and $k = -1$ find its center, its axes, its asymptotes and its eccentricity.

(iii) Calculate the equation of an ellipse for which the point $F(1,0)$ is a focus, the line $x - y = 0$ is an axis, and passes through the point $(1,1)$.

(Final exam, July 2020)

10.– Consider the family of conics dependent on the parameter $a \in \mathbb{R}$:

$$x^2 + 8xy - ay^2 - 2x - 2ay = 0$$

a) Classify these conics in terms of a.

b) For $a = -1$ find the distance between its two foci.

c) For the conics of the family that consist of a pair of intersecting lines, find such lines.

(Final exam, July 2015)

11.– Find the equations of

(a) a parabola which passes through the points $P = (0,3)$, $Q = (2,6)$ and whose axis is the line $x - y + 1 = 0$. **(Final exam, May 2016)**

(b) the conic whose center is $C(1,1)$ and such that $y = 1$ is an axis and the polar of the point $(2,2)$ is the line $x + y - 3 = 0$.

(c) the equation of an ellipse whose center is the origin, whose focus is the point $F(1,1)$ and passes through the point $(1,-1)$ **(Final exam, July 2016)**

(d) a hyperbola that passes through the origin, has the line $x - 2y - 1 = 0$ as an asymptote and one of its axes is the line $x - y - 1 = 0$. **(Extraordinary exam, September 2010)**

(e) a parabola passing through the points $P = (0,2)$, $Q = (1,0)$ and such that the line joining P and Q is the polar line of the point $(0,0)$. **(Final exam, June 2008)**

(f) a conic whose axis is the line $x - 2y = 0$, is tangent to $x = 3$ and passes through the points $(3,1)$ and $(4,1)$. **(Final Exam, July 2014)**

(g) a conic with a vertex at the point $V(1,1)$, passing through the point $(2,4)$ and such that both lines $x + y - 2 = 0$ and $x = 2$ are tangent to it. **(Final exam, June 2012)**

(h) an ellipse one of whose foci is the point $(-4,2)$, the farthest vertex from this focus is the point $(2,-1)$ and the eccentricity is $1/2$. **(Final exam, July 2011)**

(i) the parabola C such that: the line of equation $x + y - 2 = 0$ is the tangent to C at the vertex; C passes through the origin of coordinates; and the polar line of the point (2,1) with respect to C is parallel to the OX axis.

12.– Find the equation of a hyperbola knowing that its center is $(1,1)$, the point $(0,0)$ is a vertex, and it passes through the point $(4,1)$.

(Final exam, May 2022)

13.– What is the maximum number of parabolas there can be in a pencil of conics generated by two conics that are not of parabolic type?

(Second partial, June 2009)

14.– Find the equation of a hyperbola with the vertices at the points $(0, 0)$ and $V = (2, 2)$ and an asymptote perpendicular to the line $2x + y = 0$.

(Second partial, June 2015)

15.– Find the equation of a conic that passes through the point $(3, 2)$ and has the straight line $x - y = 0$ as an asymptote and $x - 2y + 1 = 0$ as an axis.

(Final exam, May 2016)

16.– Find the equation of a conic knowing that its center is the point $(1, 2)$, it is tangent to the line $x + y - 2 = 0$ at the point $(2, 0)$ and passes through the origin.

(Final exam, May 2018)

17.– Find the equation of an ellipse with center $(1, 2)$, a focus at $(2, 4)$ and also knowing that the distance between the two vertices located on the minor axis is 4.

(Final exam, July 2018)
