LINEAR ALGEBRA II — Practice 3.1-3.2

Affine geometry. (Academic year 2022–2023)

1.– In the affine space $\mathbb{R}^3$ we consider the canonical reference R and the reference

$$R' = \{(1,0,1);(1,1,0),(1,1,1),(1,0,0)\}.$$

We denote respectively by (x,y,z) and (x',y',z') the coordinates of a point with respect to the references R and R'. Find the equations of the plane $x+y-2z+1=0$ in the reference R'.

(Final exam, July 2014)

2.– In the ordinary affine Euclidean space and relative to a rectangular affine coordinate system, we consider the points $A(2,-1,1)$, $B(-1,0,3)$, the lines

$$r : \frac{x+1}{3} = \frac{y-2}{1} = \frac{z}{1}, \qquad s : \frac{x}{1} = \frac{y}{2} = \frac{z}{-3}$$

and the planes $P : 3x-2y+4z+8=0$, $Q : x+5y-6z-4=0$.

Lines should be determined by means of their continuous equations and planes by their Cartesian equations. Determine

(a) Line parallel to r that passes through A.

(b) Line through B and parallel to P and Q.

(c) Plane parallel to P that passes through A.

(d) Plane passing through B and parallel to r and s.

(e) Line perpendicular to Q and passing through A.

(f) Plane perpendicular to s and passing through B.

(g) Line that passes through A and is perpendicular to both r and s.

(h) Plane perpendicular to both P and Q and passing through B.

(i) Plane that contains r and is perpendicular to P.

(j) Line that passes through A, is perpendicular to s and parallel to Q.

(k) Line that contains B, is parallel to Q and intersects r.

(l) Line that intersects r and s and passes through B.

(m) Line parallel to the direction given by $\bar{v}(1,1,2)$ and intersecting both the lines r and s.

(n) Line that passes through A, intersects s and is perpendicular to r.

(o) Plane perpendicular to P, parallel to r and passing through A.

(p) Line which is simultaneously orthogonal to r and s.

(q) Distances from A to B, from A to r, from B to P and from r to s.

(r) Angles formed by r and s, by s and Q and by P and Q.

3.– In an affine space, give the equation of a line passing through the point $P = (1, 0, 1)$ and orthogonally intersecting the line

$$s \equiv \begin{cases} y + z = 4 \\ x + 3y = 11 \end{cases}$$

Find also the distance between P and s.

(Final exam, July 2018)

4.– In the affine space $\mathbb{R}^3$ an isosceles triangle ABC is considered, with unequal angle at vertex C. It is known that it is contained in the plane $x + y + z = 1$, $A = (2, -1, 0)$, $B = (0, -1, 2)$, and it has an area of $4\sqrt{3}$.

(i) Find the coordinates of vertex C.

(ii) Find the volume of the triangular pyramid that has triangle ABC as its base and the origin as vertex.

(iii) Find the equations of a symmetry with respect to a line that transforms point A into point B and leaves C fixed.

(Final exam, June 2020)

5.– In the affine space $\mathbb{R}^3$ we consider a scalar product whose Gram matrix with respect to the canonical basis is:

$$G_C = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$$

Find the distance between the point $(-4, 1, 0)$ and the line r of equations:

$$r \equiv \begin{cases} x + y - z = 1 \\ x - y + z = 3 \end{cases}$$

(Final exam, May 2018)

6.– In the affine three-dimensional space, obtain the equations of a rotation of 90^o with respect to the semi-axis with equation

$$(x, y, z) = (1, 0, 0) + \lambda(3, 0, 4), \qquad \lambda \in \mathbb{R},\ \lambda > 0.$$

(Final exam, May 2016)

7.– In the Euclidean affine plane $\mathbb{R}^2$ let A, B, C be the vertices of an equilateral triangle located in the half-plane $y \geq 0$, with $A = (0, 0)$ and $B = (2, 1)$.

(i) Find the coordinates of C.

(ii) Find the equations of a symmetry with respect to a line which takes the vertex A to the vertex B.

(Final exam, May 2022)

8.– In the Euclidean affine plane, find the equations of a symmetry that takes the line $x = 0$ to the line $3x + 4y - 4 = 0$. Is the solution unique?

(Final exam, May 2018)

9.– In the Euclidean affine space $\mathbb{R}^3$ consider the tetrahedron with vertices $A = (0, 0, 0)$, $B = (1, 1, 0)$, $C = (0, 0, 1)$ and $D = (1, 0, -1)$. Obtain its area and its volume.

(Final exam, May 2023)

10.– In the affine space $\mathbb{R}^2$ the points $A = (0, 0)$ and $B = (8, 6)$ are considered.

(i) Compute the implicit equation of the locus of points C in the plane that make AB the hypotenuse of a right triangle ABC.

(ii) If $P = (4, 14/3)$ is the centroid of one of the triangles indicated in the previous section, find its area and its perimeter.

(iii) Find the equations of a homothety which takes the point A to the point B and the point B to the point A.

(Exam July, May 2020)

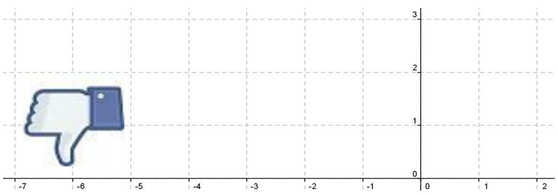
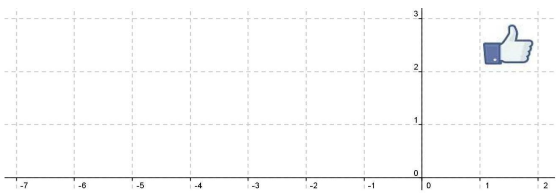
11.– In the three-dimensional affine space we consider a regular pyramid with a square base. We denote by A, B, C, D the four vertices of the base and by E the upper vertex. Knowing that the base is contained in the plane $z = 0$, that $A = (0, 0, 0)$ and $C = (4, 2, 0)$ are opposite vertices of the base and that the height of the pyramid is 5, obtain:

(i) The coordinates of the three remaining vertices.

(ii) The volume of the pyramid.

(Final exam, May 2017)

12.– Give the equation of a homothety that transforms the first figure into the second one:

(Final exam, July 2011)

13.– In the affine plane consider be the points $A(-1, 0)$ and $P(1, 0)$. Calculate the implicit equation of the locus of points B in the plane such that AB is one of the equal sides of an isosceles triangle whose orthocenter is P. What kind of curve is it?.

(Final exam, May 2015)

14.– In the affine plane consider the circumference $c : (x - 3)^2 + y^2 = 3^2$ and the line $r : y - 3 = 0$. For each line h passing through the origin, let A be the point of intersection (different from the origin) of c and h and let B be the point of intersection of r and h. Calculate the implicit equation of the locus of intersection points of the line parallel to the OX axis through A and the line parallel to the OY axis through B.

(Final exam, July 2017)

15.– In the affine plane, consider the lines $r \equiv y - 1 = 0$ and $s \equiv bex - y = 0$. For each point P on s, we draw a straight line l perpendicular to s passing through P. Let Q be the intersection point of r and l.

(i) Compute the implicit equation of the locus of all midpoints of P and Q.

(ii) Find the angle between the lines r and s.

(Final exam, May 2014)

16.– In the Euclidean affine space and relative to the canonical reference, we consider the points $A = (1, 0, 0)$ and $B = (1, 2, 2)$.

(i) Find the locus of points that are equidistant from A and B.

(ii) Find the coordinates of a point C on the plane $z = 2$, such that the triangle ABC is isosceles with AB as the unequal side and has area $2\sqrt{3}$. Is the solution unique?

(Final exam, June 2013)
