LINEAR ALGEBRA II **Exercises Unit III. Chapters 1 and 2**

Affine geometry. (Academic year 2022–2023)

1.– In the affine plane $\mathbb{R}^2$ we consider the canonical reference $R = \{(0,0);(1,0),(0,1)\}$ and another reference $R' = \{(1,2);(2,3),(3,4)\}$.

(a) If a point P has coordinates $(1,-2)_{R'}$ in reference R' calculate its coordinates in reference R.

(b) If a point Q has coordinates $(2,3)_R$ in the canonical reference, calculate its coordinates in the reference R'.

2.–

(a) In the affine plane $\mathbb{R}^2$ and with respect to the canonical reference calculate the vector, parametric, continuous and Cartesian equations of a straight line that passes through the point $(2,1)$ and has as $(1,3)$ as its direction vector.

(b) In the affine plane $\mathbb{R}^2$ and with respect to the canonical reference, calculate the vector, parametric, continuous and Cartesian equations of a line which has the equation $2x+3y-5=0$.

3.–

(a) In the affine space $\mathbb{R}^3$ and with respect to the canonical reference calculate the vector, parametric equations , continuous and Cartesian equations of a straight line that passes through the point $(2,0,-1)$ and has $(1,3,2)$ as its direction vector.

(b) In the affine space $\mathbb{R}^3$ and with respect to the canonical reference calculate the vector, parametric, continuous and Cartesian equations of a line with equations

$$\begin{aligned} x+y-2z+1=0 \\ 2x-y-z+2=0 \end{aligned}$$

4.–

(a) In the affine space $\mathbb{R}^3$ and with respect to the canonical reference calculate the vector, parametric and Cartesian equations of a plane that passes through the point $(1,0,1)$ and has $(1,0,2)$ and $(1,1,0)$ as its direction vectors.

(b) In the affine space $\mathbb{R}^3$ and with respect to the canonical reference calculate the vector, parametric and Cartesian equations of a plane that has the equation $x+2y-z-2=0$.

5.– In the Euclidean affine plane $\mathbb{R}^2$ and with respect to a rectangular reference the following lines are considered:

$$r \equiv x+2y-4=0 \qquad s \equiv x+y-3=0$$

(a) Find the angle between the lines r and s.

(b) Find the distance from the point $(1,3)$ to the line r.

6.– In the Euclidean affine space $\mathbb{R}^3$ and with respect to a rectangular reference, the following affine subspaces are considered:

$$\pi \equiv x + y - 2z + 1 = 0, \qquad r \equiv (x, y, z) = (1, 2, 3) + t(0, 1, 0), \qquad s \equiv (x, y, z) = (0, 1, 1) + s(1, 1, 2)$$

(a) Find the angle formed by π and r.

(b) Find the distance from the point $(1, 1, 0)$ to the plane π.

(c) Find the distance between lines r and s.

7.– In the Euclidean affine plane $\mathbb{R}^2$, under the usual conditions and with respect to the canonical reference, calculate the equations of:

(a) a translation with respect to the vector $\vec{u} = (2, 3)$.

(b) a homothety with center $(1, 2)$ and ratio 4.

(c) a rotation with center $(1, -2)$ and angle 30 degrees.

(d) a symmetry with respect to the line $x + y = 1$.

8.– In the Euclidean affine plane $\mathbb{R}^3$, under the usual conditions and with respect to the canonical reference, calculate the equations of:

(a) a translation with respect to the vector $\vec{u} = (1, 0, 2)$.

(b) a homothety with center $(1, 2, -1)$ and ratio -2.

(c) a rotation of angle 45 degrees with respect to the semi-axis $(x, y, z) = (1, 0, 0) + t(0, 1, 0)$, $t > 0$.

(d) a symmetry with respect to the plane $x + y - z = 1$.

Solutions .

---

1. (a) $(-3, -3)$. (b) $(-1, 1)$.

2. (a) Vector: $(x, y) = (2, 1) + t(1, 3)$. Parametric: $\begin{cases} x = 2 + t \\ y = 1 + 3t \end{cases}$ Continuous: $\dfrac{x-2}{1} = \dfrac{y-1}{3}$. Cartesian: $3x - y - 5 = 0$. (b) Vector: $(x, y) = (1, 1) + t(3, -2)$. Parametric: $\begin{cases} x = 1 + 3t \\ y = 1 - 2t \end{cases}$ Continuous: $\dfrac{x-1}{3} = \dfrac{y-1}{-2}$. Cartesian: $2x + 3y - 5 = 0$.

3. (a) Vector: $(x, y, z) = (2, 0, -1) + t(1, 3, 2)$. Parametric: $\begin{cases} x = 2 + t \\ y = 3t \\ z = -1 + 2t \end{cases}$ Continuous: $\dfrac{x-2}{1} = \dfrac{y}{3} = \dfrac{z+1}{2}$. Cartesian: $3xy - 6 = 0$, $2y - 3z - 3 = 0$. (b) Vectorial: $(x, y, z) = (-1, 0, 0) + t(1, 1, 1)$. Parametric: $\begin{cases} x = -1 + t \\ y = t \\ z = t \end{cases}$ Continuous: $\dfrac{x+1}{1} = \dfrac{y}{1} = \dfrac{z}{1}$. Cartesians: $x + y - 2z + 1 = 0$, $2x - y - z + 2 = 0$.

4. (a) Vector: $(x, y, z) = (1, 0, 1) + t(1, 0, 2) + s(1, 1, 0)$. Parametric: $\begin{cases} x = 1 + t + s \\ y = s \\ z = 1 + 2t \end{cases}$ Cartesian: $2x - 2y - z - 1 = 0$. (b) Vector: $(x, y, z) = (2, 0, 0) + t(1, 0, 1) + s(2, -1, 0)$. Parametric: $\begin{cases} x = 2 + t + 2s \\ y = -s \\ z = t \end{cases}$

Cartesian: $x + 2y - z - 2 = 0$.

5. (a) $arccos(3/\sqrt{10})$. (b) $3/\sqrt{5}$

6. (a) $arcsin(1/\sqrt{6})$. (b) $3/\sqrt{6}$. (c) 0 (the lines intersect at the point $(1, 2, 3)$)

7. (a) $f(x, y) = (x+2, y+3)$ (b) $f(x, y) = (4x-3, 4y-6)$. (c) $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} + \begin{pmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{pmatrix} \begin{pmatrix} x-1 \\ y+2 \end{pmatrix}$.
(d) $f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x-1 \\ y \end{pmatrix}$.

8. (a) $f(x, y, z) = (x + 1, y, z + 2)$ (b) $f(x, y) = (3 - 2x, 6 - 2y, -3 - 2z)$. (c) $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ -1/\sqrt{2} & 0 & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} x-1 \\ y \\ z \end{pmatrix}$. (d) $f\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 1 & -2 & 2 \\ -2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix} \begin{pmatrix} x-1 \\ y \\ z \end{pmatrix}$.