Linear Algebra II. Academic Year 2022-2023. Quadratic forms. Voluntary assignment.

Let $c_1c_2c_3c_4c_5c_6c_7c_8$ be the eight digits of your DNI number[1]. For instance, if your DNI number is 32478910, then $c_1 = 3$, $c_2 = 2$, $c_3 = 4$, $c_4 = 7$, $c_5 = 8$, $c_6 = 9$, $c_7 = 1$, $c_8 = 0$.

Let a be the integer part of $c_5/2$.

Given $k \in \mathbb{R}$ we consider the quadratic form $w : \mathbb{R}^3 \to \mathbb{R}$,

$$w(x, y, z) = x^2 - a^2z^2 + 4xy + 2xz + 2kyz.$$

1. Classify the quadratic form in terms of k, indicating its rank and signature.
2. Obtain a basis of conjugate vectors.
3. For the cases where the quadratic form is indefinite and of rank 2, give the implicit equation with respect to the canonical basis of the two planes into which the set of self-conjugate vectors can be decomposed.
4. For $k = 2$ give the implicit equations of a subspace $U \subset \mathbb{R}^3$ of the maximum possible dimension, such that $w(\vec{u}) < 0$ for every nonzero $\vec{u} \in U$.
5. For $k = 4$ give the implicit equations of a subspace $V \subset \mathbb{R}^3$ of the maximum possible dimension, such that $w(\vec{v}) > 0$ for every nonzero $\vec{v} \in U$.
6. For $k = 0$ give nonzero vectors $\vec{u}, \vec{v}, \vec{w}$ such that $w(\vec{u}) > 0$, $w(\vec{v}) = 0$ and $w(\vec{w}) < 0$.
7. A function $w(x, y, z)$ is said to be bounded from below (respectively from above) if there exists some constant M such that $w(x, y, z) > M$ (respectively $w(x, y, z) < M$) for all vectors $(x, y, z) \in \mathbb{R}^3$. Is there any value of k for which w is bounded from above or from below?

Rules:

- The submission of the assignment is voluntary.

- The deadline is Friday, March 3 at 11:59 p.m.

- It will contribute a maximum of 0.5 points towards the final mark of the subject, as explained in the introductory class.

- **Only the assignments submitted on time will be considered**.

- Any indication of academic malpractice will result in disciplinary action, including not passing the course.

- In the submitted assignment you must include your name and DNI, and **keep a minimum of quality in the presentation**.

- The assignment should be submitted in PDF format through the Teams platform. **The name of the file must be "TT1-Name and surname.pdf". For example: "TT1-Luis Fuentes García.pdf"**. They will also be accepted in paper form exceptionally.

- Students may be required to present and explain the submitted assignment orally and show full knowledge of what they have written.

[1] If the identification document has less than 8 digits, you can substitute the letters for the number 5. For example if it is $ZZ013456$ you can use 55013456.