Let $c_1c_2c_3c_4c_5c_6c_7c_8$ be the eight digits of your DNI[(1)]. For example, if the DNI is 32478910, then $c_1 = 3$, $c_2 = 2$, $c_3 = 4$, $c_4 = 7$, $c_5 = 8$, $c_6 = 9$, $c_7 = 1$, $c_8 = 0$.

For each i, with $1 \le i \le 8$ we call a_i the remainder of c_i module 3, that is, the remainder obtained by dividing c_i by 3. In the above example $a_1 = 0$, $a_2 = 2$, $a_3 = 1$, $a_4 = 1$, $a_5 = 2$, $a_6 = 0$, $a_7 = 1$, $a_8 = 0$.

Let $\mathcal{M}_{2\times 2}(\mathbb{R})$ be the vector space of 2×2 square matrices. Consider the following subsets:

$$U = \left\{A \in \mathcal{M}_{2\times 2}(\mathbb{R}) | A = A^t\right\}$$

$$V = \mathcal{L}\left\{\begin{pmatrix} 1 & 0 \\ a_1 & a_2 \end{pmatrix}, \begin{pmatrix} a_3 & 0 \\ a_4 & 1 \end{pmatrix}, \begin{pmatrix} a_3 & 0 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ a_6 & a_5 \end{pmatrix}\right\}$$

$$W = \{A \in \mathcal{M}_{2\times 2}(\mathbb{R}) | det(A) = 0\}$$

$$Z = \mathcal{L}\left\{\begin{pmatrix} a_1 & 1 \\ 2 & a_3 \end{pmatrix}\right\}$$

1. Which of them are vector subspaces of $\mathcal{M}_{2\times 2}(\mathbb{R})$?. Justify the answer.
2. Find the parametric and implicit equations of $U \cap V$ with respect to the canonical basis.
3. Find $dim(U), dim(V), dim(U \cap V), dim(U + V)$. Are U and V complementary subspaces?
4. Can any matrix of $\mathcal{M}_{2\times 2}(\mathbb{R})$ be written as the sum of a matrix in U and another matrix in V? Is there a matrix that can be expressed in two different ways as the sum of one of U and another of V? If yes, give an example.
5. Prove that $B = \left\{\begin{pmatrix} 1 & 0 \\ 0 & a_2 \end{pmatrix}, \begin{pmatrix} 0 & 2 \\ a_6 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & a_7 \end{pmatrix}\right\}$ is a basis of $\mathcal{M}_{2\times 2}(\mathbb{R})$.
6. Find the parametric and implicit equations of Z with respect to the basis B.
7. Study whether U and Z are complementary subspaces. If they are, find the matrix $A \in \mathcal{M}_{2\times 2}(\mathbb{R})$ which is the projection of $\begin{pmatrix} a_5 & 2 \\ 0 & a_2 \end{pmatrix}$ onto Z along U and the projection of the same matrix onto U along Z.
8. Give the implicit equations in the canonical basis of a vector subspace T such that T and V are complementary subspaces.

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(1) In the event that the DNI has less than 8 digits, you can complete it on the left with as many "fours" as necessary. For example if it is $ZZ13456$ you can use 44413456.

Rules:

- The submission of the assignment is voluntary.

- The deadline is Friday, December 16 at 11:59 p.m.

- It will contribute a maximum of 0.5 points towards the final mark of the subject, as explained in the introductory class.

- **Only the assignments submitted on time will be considered**.

- Any indication of academic malpractice will result in disciplinary action, including not passing the course.

- In the submitted assignment you must include your name and DNI, and **keep a minimum of quality in the presentation**.

- The assignment should be submitted in PDF format through the Teams platform. **The name of the file must be "T2-Name and surname.pdf". For example: "T2-Luis Fuentes García.pdf"**. They will also be accepted in paper form exceptionally.

- Students may be required to present and explain the submitted assignment orally and show full knowledge of what they have written.