Let d_1 and d_2 be the last two digits of your DNI. Throughout the problem we will define $a = 2 + [d_1/2]$ and $b = 2 + [d_2/2]$, where $[x]$ is the floor function, that is, the greatest integer less than or equal to x.

The goal of this assigment is counting how many passwords of a given length can be formed by combining the first a letters of the alphabet and the first b digits $(0, 1, \ldots, b - 1)$, under certain conditions.

1. How many 7-character passwords (letters and digits) can be formed?

2. How many 7-character passwords can be formed such that letters and digits alternate (there can't be two letters or two digits in a row)?

3. Generalize the previous result for an arbitrary n-character password.

4. How many 7-character passwords can be formed that contain 3 letters and 4 digits?

5. Now, we consider the n-character passwords where there are no two consecutive letters (but there can be two consecutive digits). Let x_n be the number of such passwords with a letter as its last character and y_n those with a digit as its last character.

   (a) Justify that $x_{n+1} = a \cdot y_n$ and $y_{n+1} = b \cdot (x_n + y_n)$.

   (b) Express the previous relation with a matrix equation, finding $A \in \mathcal{M}_{2 \times 2}$ such that:
   $$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = A \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

   (c) Prove by induction that:
   $$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = A^{n-1} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

   (d) Using the previous expressions, compute the number of 9-character passwords such that there are no consecutive letters.

Answers must be reasonably justified.