

Let  $d_1$  and  $d_2$  be the last two digits of your *DNI*. Throughout the problem we will define  $a = 2 + \lfloor d_1/2 \rfloor$  and  $b = 2 + \lfloor d_2/2 \rfloor$ , where  $\lfloor x \rfloor$  is the floor function, that is, the greatest integer less than or equal to  $x$ .

The goal of this assignment is counting how many passwords of a given length can be formed by combining the first  $a$  letters of the alphabet and the first  $b$  digits  $(0, 1, \dots, b - 1)$ , under certain conditions.

1. How many 7-character passwords (letters and digits) can be formed?
2. How many 7-character passwords can be formed such that letters and digits alternate (there can't be two letters or two digits in a row)?
3. Generalize the previous result for an arbitrary  $n$ -character password.
4. How many 7-character passwords can be formed that contain 3 letters and 4 digits?
5. Now, we consider the  $n$ -character passwords where there are no two consecutive letters (but there can be two consecutive digits). Let  $x_n$  be the number of such passwords with a letter as its last character and  $y_n$  those with a digit as its last character.

(a) Justify that  $x_{n+1} = a \cdot y_n$  and  $y_{n+1} = b \cdot (x_n + y_n)$ .

(b) Express the previous relation with a matrix equation, finding  $A \in \mathcal{M}_{2 \times 2}$  such that:

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = A \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

(c) Prove by induction that:

$$\begin{pmatrix} x_n \\ y_n \end{pmatrix} = A^{n-1} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

(d) Using the previous expressions, compute the number of 9-character passwords such that there are no consecutive letters.

**Rules:**

- The submission of the assignment is voluntary.
- The deadline is Friday, October 28 at 11:59 p.m.
- It will contribute a maximum of 0.5 points towards the final mark of the subject, as explained in the introductory class.
- **Only the assignments submitted on time will be considered.**
- Any indication of academic malpractice will result in disciplinary action, including not passing the course.
- In the submitted assignment you must include your name and *DNI*, and **keep a minimum of quality in the presentation.**
- The assignment should be submitted in PDF format through the Teams platform. However, they will also be accepted in paper form exceptionally.
- Students may be required to present and explain the submitted assignment orally and show full knowledge of what they have written.

Answers must be reasonably justified.