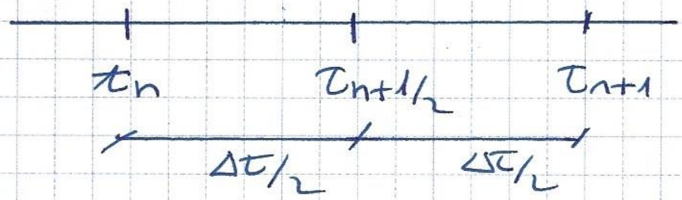
Aproximación de $f(\tau_{n+1/2})$ en función de $f(\tau_n)$, $f(\tau_{n+1})$



$$\begin{cases} \tau_{n+1/2} = \tau_n + \Delta\tau/2 \\ \tau_{n+1} = \tau_n + \Delta\tau \end{cases}$$

Desarrollemos $f(t)$ en Serie de Taylor entorno a $\tau_{n+1/2}$:

$$\begin{cases} f_{n+1} = f_{n+1/2} + f'_{n+1/2}\dfrac{\left(\frac{\Delta\tau}{2}\right)}{1!} + f''_{n+1/2}\dfrac{\left(\Delta\tau/2\right)^2}{2!} + f'''_{n+1/2}\dfrac{\left(\Delta\tau/2\right)^3}{3!} + \Theta\left(\left(\Delta\tau/2\right)^4\right) \\ f_n = f_{n+1/2} - f'_{n+1/2}\dfrac{\left(\Delta\tau/2\right)}{1!} + f''_{n+1/2}\dfrac{\left(\Delta\tau/2\right)^2}{2!} - f'''_{n+1/2}\dfrac{\left(\Delta\tau/2\right)^3}{3!} + \Theta\left(\left(\Delta\tau/2\right)^4\right) \end{cases}$$

$$\begin{aligned} \varphi f_{n+1} + (1-\varphi) f_n = & \left(\varphi + (1-\varphi)\right) f_{n+1/2} \\ & + \left(\varphi - (1-\varphi)\right) f'_{n+1/2}\frac{\left(\Delta\tau/2\right)}{1!} \\ & + \left(\varphi + (1-\varphi)\right) f''_{n+1/2}\frac{\left(\Delta\tau/2\right)^2}{2!} \\ & + \left(\varphi - (1-\varphi)\right) f'''_{n+1/2}\frac{\left(\Delta\tau/2\right)^3}{3!} \\ & + \Theta\left(\left(\Delta\tau/2\right)^4\right) \end{aligned}$$

Luego:

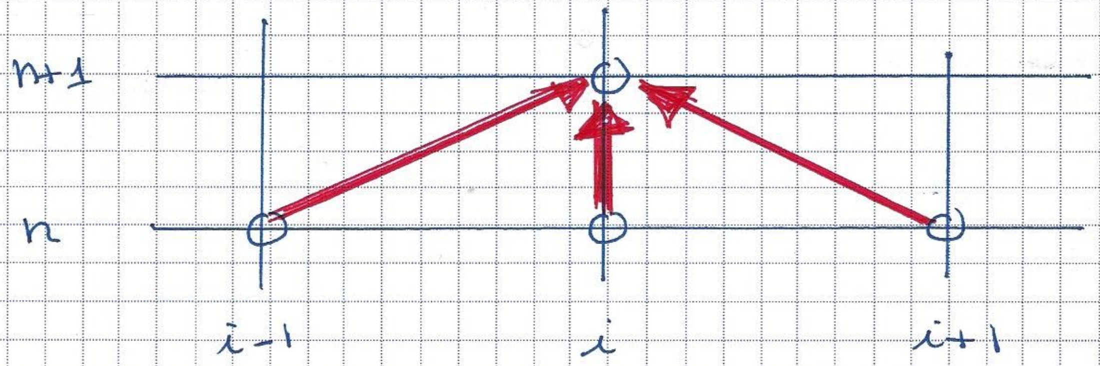
$$\boxed{f_{n+1/2} = \left[\varphi f_{n+1} + (1-\varphi) f_n\right] - \frac{(2\varphi-1)}{2} f'_{n+1/2}\,\Delta\tau + \Theta(\Delta\tau^2)}$$

Para $\varphi = 1/2 \Rightarrow f_{n+1/2} = \dfrac{f_n + f_{n+1}}{2} + \Theta(\Delta\tau^2)$

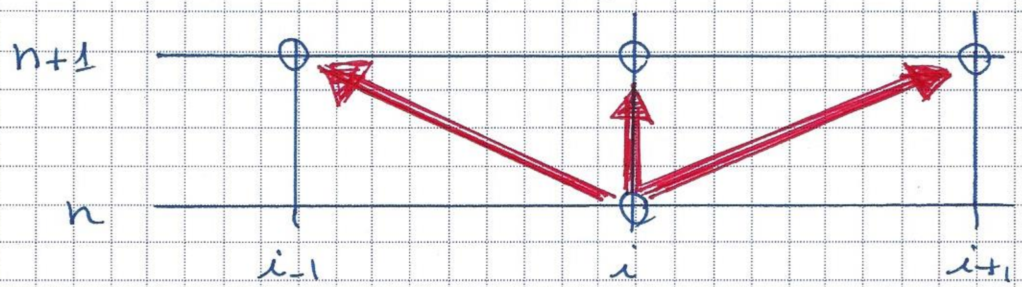
Nota: $f_n = f(\tau_n)$, $f_{n+1/2} = f(\tau_{n+1/2})$, $f_{n+1} = f(\tau_{n+1})$

Propagación de Información

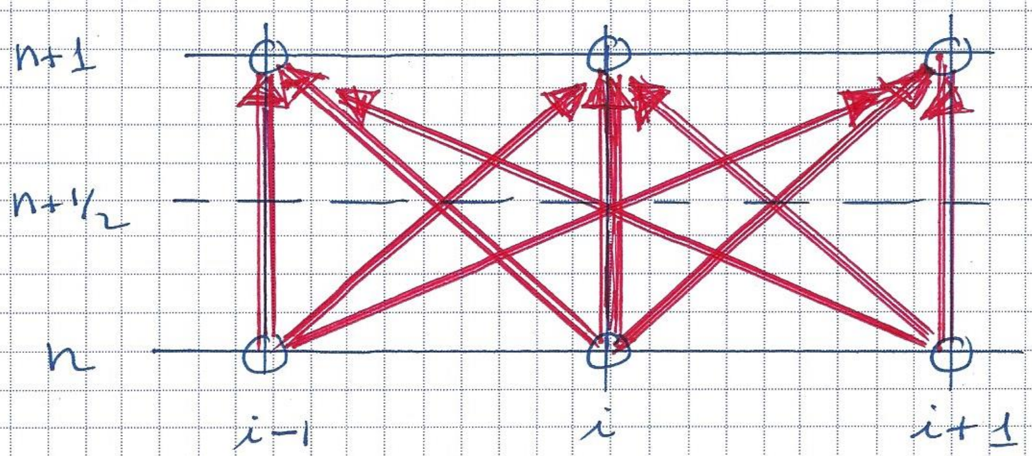
Método Explícito



Método Implícito



Método de Crank-Nicolson



Reflexión: ¿Hasta dónde llega la información aportada por las C.I. y las C.C.?

¿Qué parte de la malla no se ve afectada?