1. MÉTODOS DE INTERVALO SIMPLE

1.1. Métodos basados en la aproximación de la derivada

1.1.1. Método de Euler

$$y_{i+1} = y_i + h\varphi(x_i, y_i) \qquad ;\tau(h)$$

1.1.2 Método de Diferencias Centradas

$$y_{i+1} = y_{i-1} + 2h\varphi(x_i, y_i) \qquad ;\tau(h^2)$$

1.2. Métodos basados en desarrollos en serie

1.2.1. Método del Desarrollo en Serie de Segundo Orden

$$y_{i+1} = y_i + h\varphi(x_i, y_i) + \frac{h^2}{2}\left(\varphi'_x(x_i, y_i) + \varphi'_y(x_i, y_i)\varphi(x_i, y_i)\right) \qquad ;\tau(h^2)$$

1.3. Métodos de Runge-Kutta

1.3.1. Métodos de Runge-Kutta de Segundo Orden

$$\begin{aligned} y_{i+1} = y_i + h\Phi(x_i, y_i) \\ \Phi(x, y) =& w_0 k_0 + w_1 k_1 \\ k_0 =& \varphi(x, y) \\ k_1 =& \varphi(x + \theta_1 h, y + (w_{10} k_0) h) \end{aligned}$$

$$w_0 + w_1 = 1; \qquad w_1\theta_1 = \frac{1}{2}; \qquad w_1 w_{10} = \frac{1}{2}$$

1.3.1.1. Método de Euler Modificado

$$w_0 = 0; \qquad w_1 = 1; \qquad \theta_1 = \frac{1}{2}; \qquad w_{10} = \frac{1}{2} \qquad ;\tau(h^2)$$

1.3.1.2. Método de Heun

$$w_0 = \frac{1}{2}; \qquad w_1 = \frac{1}{2}; \qquad \theta_1 = 1; \qquad w_{10} = 1 \qquad ;\tau(h^2)$$

1.3.1.3. Método de Ralston

$$w_0 = \frac{1}{3}; \qquad w_1 = \frac{2}{3}; \qquad \theta_1 = \frac{3}{4}; \qquad w_{10} = \frac{3}{4} \qquad ;\tau(h^2)$$

1.3.1.4. Método de Tercer Orden para $\varphi'_y = 0$

$$w_0 = \frac{1}{4}; \qquad w_1 = \frac{3}{4}; \qquad \theta_1 = \frac{2}{3}; \qquad w_{10} = \frac{2}{3} \qquad ;\tau(h^2)$$

1.3.2. Métodos de Runge-Kutta de Tercer Orden

$$\begin{aligned}
y_{i+1} = y_i + h\Phi(x_i, y_i)& \\
\Phi(x,y) =& w_0k_0 + w_1k_1 + w_2k_2 \\
k_0 =& \varphi(x,y) \\
k_1 =& \varphi(x + \theta_1 h, y + (w_{10}k_0)h) \\
k_2 =& \varphi(x + \theta_2 h, y + (w_{20}k_0 + w_{21}k_1)h)
\end{aligned}$$

$$\begin{aligned}
w_0 + w_1 + w_2 =& 1; & w_1\theta_1 + w_2\theta_2 =& \frac{1}{2}; & w_1\theta_1^2 + w_2\theta_2^2 =& \frac{1}{3} \\
w_2\theta_1 w_{21} =& \frac{1}{6}; & \theta_1 =& w_{10}; & \theta_2 =& w_{20} + w_{21}
\end{aligned}$$

1.3.2.1. Método de Kutta

$$\begin{aligned}
w_0 =& \frac{1}{6}; & w_1 =& \frac{4}{6}; & w_2 =& \frac{1}{6} \\
\theta_1 =& \frac{1}{2}; & \theta_2 =& 1; & & \\
w_{10} =& \frac{1}{2}; & w_{20} =& -1; & w_{21} =& 2
\end{aligned} \qquad ;\tau(h^3)$$

1.3.2.2. Método de Heun de Tercer Orden

$$\begin{aligned}
w_0 =& \frac{1}{4}; & w_1 =& 0; & w_2 =& \frac{3}{4} \\
\theta_1 =& \frac{1}{3}; & \theta_2 =& \frac{2}{3}; & & \\
w_{10} =& \frac{1}{3}; & w_{20} =& 0; & w_{21} =& \frac{2}{3}
\end{aligned} \qquad ;\tau(h^3)$$

1.3.3. Métodos de Runge-Kutta de Cuarto Orden

$$
\begin{aligned}
y_{i+1} = y_i + h\Phi(x_i, y_i) \\
\Phi(x,y) =& w_0 k_0 + w_1 k_1 + w_2 k_2 + w_3 k_3 \\
k_0 =& \varphi(x,y) \\
k_1 =& \varphi(x+\theta_1 h, y + (w_{10}k_0)h) \\
k_2 =& \varphi(x+\theta_2 h, y + (w_{20}k_0 + w_{21}k_1)h) \\
k_3 =& \varphi(x+\theta_3 h, y + (w_{30}k_0 + w_{31}k_1 + w_{32}k_2)h)
\end{aligned}
$$

1.3.3.1. Método de Kutta de Cuarto Orden

$$
\left.
\begin{aligned}
w_0 =& \frac{1}{6}; & w_1 =& \frac{1}{3}; & w_2 =& \frac{1}{3}; & w_3 =& \frac{1}{6} \\
\theta_1 =& \frac{1}{2}; & \theta_2 =& \frac{1}{2}; & \theta_3 =& 1 & & \\
w_{10} =& \frac{1}{2}; & w_{20} =& 0; & w_{21} =& \frac{1}{2} & & \\
w_{30} =& 0; & w_{31} =& 0; & w_{32} =& 1 & &
\end{aligned}
\right. ;\tau(h^4)
$$

1.3.3.2. Método de cuarto orden asociado a la cuadratura de Newton-Cotes

$$
\left.
\begin{aligned}
w_0 =& \frac{1}{8}; & w_1 =& \frac{3}{8}; & w_2 =& \frac{3}{8}; & w_3 =& \frac{1}{8} \\
\theta_1 =& \frac{1}{3}; & \theta_2 =& \frac{2}{3}; & \theta_3 =& 1 & & \\
w_{10} =& \frac{1}{3}; & w_{20} =& -\frac{1}{3}; & w_{21} =& 1 & & \\
w_{30} =& 1; & w_{31} =& -1; & w_{32} =& 1 & &
\end{aligned}
\right. ;\tau(h^4)
$$

1.3.3.3. Método de Gill

$$
\left.
\begin{aligned}
w_0 =& \frac{1}{6}; & w_1 =& \frac{2}{6}(1-\frac{1}{\sqrt{2}}); & w_2 =& \frac{2}{6}(1+\frac{1}{\sqrt{2}}); & w_3 =& \frac{1}{6} \\
\theta_1 =& \frac{1}{2}; & \theta_2 =& \frac{1}{2}; & \theta_3 =& 1 & & \\
w_{10} =& \frac{1}{2}; & w_{20} =& (-\frac{1}{2}+\frac{1}{\sqrt{2}}); & w_{21} =& (1-\frac{1}{\sqrt{2}}) & & \\
w_{30} =& 0; & w_{31} =& -\frac{1}{\sqrt{2}}; & w_{32} =& (1+\frac{1}{\sqrt{2}}) & &
\end{aligned}
\right. ;\tau(h^4)
$$

2. MÉTODOS DE INTERVALO MULTIPLE

2.1. Fórmulas Abiertas (PREDICTORES) (Métodos de Adams-Bashford)

2.1.1. k=0, r=3

$$y_{i+1} = y_i + \frac{h}{24}(55\varphi_i - 59\varphi_{i-1} + 37\varphi_{i-2} - 9\varphi_{i-3}) \qquad ;\tau(h^4)$$

2.1.2. k=1, r=1

$$y_{i+1} = y_{i-1} + 2h\varphi_i \qquad ;\tau(h^2)$$

2.1.3. k=3, r=3

$$y_{i+1} = y_{i-3} + \frac{4h}{3}(2\varphi_i - \varphi_{i-1} + 2\varphi_{i-2}) \qquad ;\tau(h^4)$$

2.1.4. k=5, r=5

$$y_{i+1} = y_{i-5} + \frac{3h}{10}(11\varphi_i - 14\varphi_{i-1} + 26\varphi_{i-2} - 14\varphi_{i-3} + 11\varphi_{i-4}) \qquad ;\tau(h^6)$$

2.2. Fórmulas Cerradas (CORRECTORES) (Métodos de Moulton)

2.2.1. k=0, r=3

$$y_{i+1} = y_i + \frac{h}{24}(9\varphi_{i+1} + 19\varphi_i - 5\varphi_{i-1} + \varphi_{i-2}) \qquad ;\tau(h^4)$$

2.2.2. k=1, r=3

$$y_{i+1} = y_{i-1} + \frac{h}{3}(\varphi_{i+1} + 4\varphi_i + \varphi_{i-1}) \qquad ;\tau(h^4)$$

2.2.3. k=3, r=5

$$y_{i+1} = y_{i-3} + \frac{2h}{45}(7\varphi_{i+1} + 32\varphi_i + 12\varphi_{i-1} + 32\varphi_{i-2} + 7\varphi_{i-3}) \qquad ;\tau(h^6)$$

2.3. Métodos PREDICTOR-CORRECTOR

2.3.1. Método de Adams-Moulton de Cuarto Orden

Predictor: k=0, r=3
Corrector: k=0, r=3

2.3.2. Método de Milne de Cuarto Orden

Predictor: k=3, r=3
Corrector: k=1, r=3

2.3.3. Método de Milne de Sexto Orden

Predictor: k=5, r=5
Corrector: k=3, r=5