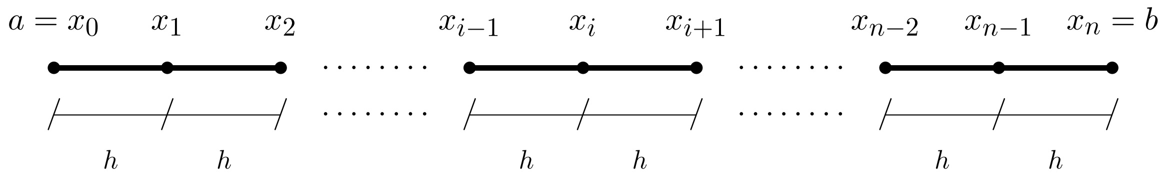
CUADRATURAS CERRADAS DE NEWTON-COTES



$$h = \frac{b-a}{n}, \qquad \xi \in [a,b], \qquad x_i = a + i\,h, \qquad f_i = f(x_i), \qquad i = 0, \dots, n.$$

$$n = 1 \longrightarrow \int_a^b f(x)dx = \frac{h}{2} \Big(f_0 + f_1\Big) - \frac{1}{12}\, h^3\, f^{(2)}(\xi) \qquad \text{[TRAPECIO]}$$

$$n = 2 \longrightarrow \int_a^b f(x)dx = \frac{h}{3} \Big((f_0 + f_2) + 4f_1\Big) - \frac{1}{90}\, h^5\, f^{(4)}(\xi) \qquad \text{[SIMPSON]}$$

$$n = 3 \longrightarrow \int_a^b f(x)dx = \frac{3h}{8} \Big((f_0 + f_3) + 3(f_1 + f_2)\Big) - \frac{3}{80}\, h^5\, f^{(4)}(\xi) \qquad \text{[2.ª de Simpson]}$$

$$n = 4 \longrightarrow \int_a^b f(x)dx = \frac{2h}{45} \Big(7(f_0 + f_4) + 32(f_1 + f_3) + 12f_2\Big) - \frac{8}{945}\, h^7\, f^{(6)}(\xi) \qquad \text{[Boole]}$$

$$n = 5 \longrightarrow \int_a^b f(x)dx = \frac{5h}{288} \Big(19(f_0 + f_5) + 75(f_1 + f_4) + 50(f_2 + f_3)\Big) - \frac{275}{12096}\, h^7\, f^{(6)}(\xi)$$

$$n = 6 \longrightarrow \int_a^b f(x)dx = \frac{h}{140} \Big(41(f_0 + f_6) + 216(f_1 + f_5) + 27(f_2 + f_4) + 272f_3\Big) - \frac{9}{1400}\, h^9\, f^{(8)}(\xi)$$

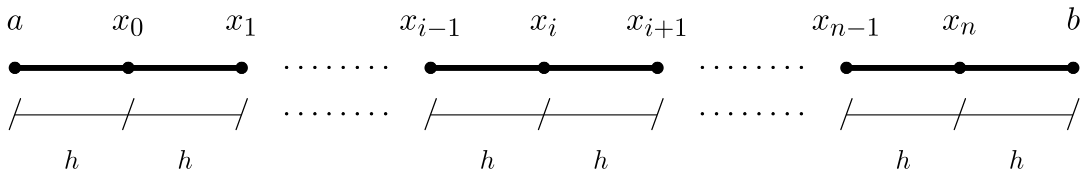
$$n = 7 \longrightarrow \int_a^b f(x)dx = \frac{7h}{17280} \Big(751(f_0 + f_7) + 3577(f_1 + f_6) + 1323(f_2 + f_5) + 2989(f_3 + f_4)\Big) - \frac{8183}{518400}\, h^9\, f^{(8)}(\xi)$$

$$n = 8 \longrightarrow \int_a^b f(x)dx = \frac{4h}{14175} \Big(989(f_0 + f_8) + 5888(f_1 + f_7) - 928(f_2 + f_6) + 10496(f_3 + f_5) - 4540f_4\Big) - \frac{2368}{467775}\, h^{11}\, f^{(10)}(\xi)$$

$$n = 9 \longrightarrow \int_a^b f(x)dx = \frac{9h}{89600} \Big(2857(f_0 + f_9) + 15741(f_1 + f_8) + 1080(f_2 + f_7) + 19344(f_3 + f_6) + 5778(f_4 + f_5)\Big) - \frac{173}{14620}\, h^{11}\, f^{(10)}(\xi)$$

$$n = 10 \longrightarrow \int_a^b f(x)dx = \frac{5h}{299376} \Big(16067(f_0 + f_{10}) + 106300(f_1 + f_9) - 48525(f_2 + f_8) + 272400(f_3 + f_7) - 260550(f_4 + f_6) + 427368f_5\Big) - \frac{1346350}{326918592}\, h^{13}\, f^{(12)}(\xi)$$

CUADRATURAS ABIERTAS DE NEWTON-COTES



$$h = \frac{b-a}{n+2}, \qquad \xi \in [a,b], \qquad x_i = a + (i+1)\,h, \qquad f_i = f(x_i), \qquad i = 0, \dots, n.$$

$$n = 0 \longrightarrow \int_a^b f(x)dx = 2h \Big(f_0\Big) + \frac{1}{3}\, h^3\, f^{(2)}(\xi)$$

$$n = 1 \longrightarrow \int_a^b f(x)dx = \frac{3h}{2} \Big(f_0 + f_1\Big) + \frac{3}{4}\, h^3\, f^{(2)}(\xi)$$

$$n = 2 \longrightarrow \int_a^b f(x)dx = \frac{4h}{3} \Big(2(f_0 + f_2) - f_1\Big) + \frac{28}{90}\, h^5\, f^{(4)}(\xi)$$

$$n = 3 \longrightarrow \int_a^b f(x)dx = \frac{5h}{24} \Big(11(f_0 + f_3) + (f_1 + f_2)\Big) + \frac{95}{144}\, h^5\, f^{(4)}(\xi)$$

$$n = 4 \longrightarrow \int_a^b f(x)dx = \frac{6h}{20} \Big(11(f_0 + f_4) - 14(f_1 + f_3) + 26 f_2\Big) + \frac{41}{140}\, h^7\, f^{(6)}(\xi)$$

$$n = 5 \longrightarrow \int_a^b f(x)dx = \frac{7h}{1440} \Big(611(f_0 + f_5) - 453(f_1 + f_4) + 562(f_2 + f_3)\Big) + \frac{5257}{8640}\, h^7\, f^{(6)}(\xi)$$

$$n = 6 \longrightarrow \int_a^b f(x)dx = \frac{8h}{945} \Big(460(f_0 + f_6) - 954(f_1 + f_5) + 2196(f_2 + f_4) - 2459 f_3\Big) + \frac{3956}{14175}\, h^9\, f^{(8)}(\xi)$$

CUADRATURAS DE GAUSS-LEGENDRE

$$\int_{-1}^{+1} f(z)\,dz = \sum_{i=0}^{n} \omega_i\, f(z_i) + E_n, \qquad E_n = K_n\; f^{(2n+2)}(\xi), \quad \text{con} \begin{cases} K_n = \dfrac{2^{2n+3}((n+1)!)^4}{(2n+3)((2n+2)!)^3}, \\ \xi \in [-1,+1]. \end{cases}$$

$$\int_a^b F(x)\,dx = \int_{-1}^{+1} f(z)\,dz, \qquad \text{donde} \begin{cases} x = \left(\dfrac{b+a}{2}\right) + \left(\dfrac{b-a}{2}\right)\, z, \\ f(z) = \left(\dfrac{b-a}{2}\right)\; F(x). \end{cases}$$

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n	i	z_i	ω_i
0	0	0.00000000000000E+00	0.20000000000000E+01
1	0	-0.57735026918963E+00	0.10000000000000E+01
	1	0.57735026918963E+00	0.10000000000000E+01
2	0	-0.77459666924148E+00	0.55555555555556E+00
	1	0.00000000000000E+00	0.88888888888889E+00
	2	0.77459666924148E+00	0.55555555555556E+00
3	0	-0.86113631159405E+00	0.34785484513745E+00
	1	-0.33998104358486E+00	0.65214515486255E+00
	2	0.33998104358486E+00	0.65214515486255E+00
	3	0.86113631159405E+00	0.34785484513745E+00
4	0	-0.90617984593866E+00	0.23692688505619E+00
	1	-0.53846931010568E+00	0.47862867049937E+00
	2	0.00000000000000E+00	0.56888888888889E+00
	3	0.53846931010568E+00	0.47862867049937E+00
	4	0.90617984593866E+00	0.23692688505619E+00
5	0	-0.93246951420315E+00	0.17132449237917E+00
	1	-0.66120938646626E+00	0.36076157304814E+00
	2	-0.23861918608320E+00	0.46791393457269E+00
	3	0.23861918608320E+00	0.46791393457269E+00
	4	0.66120938646626E+00	0.36076157304814E+00
	5	0.93246951420315E+00	0.17132449237917E+00
6	0	-0.94910791234276E+00	0.12948496616887E+00
	1	-0.74153118559939E+00	0.27970539148928E+00
	2	-0.40584515137740E+00	0.38183005050512E+00
	3	0.00000000000000E+00	0.41795918367347E+00
	4	0.40584515137740E+00	0.38183005050512E+00
	5	0.74153118559939E+00	0.27970539148928E+00
	6	0.94910791234276E+00	0.12948496616887E+00
7	0	-0.96028985649754E+00	0.10122853629038E+00
	1	-0.79666647741363E+00	0.22238103445337E+00
	2	-0.52553240991633E+00	0.31370664587789E+00
	3	-0.18343464249565E+00	0.36268378337836E+00
	4	0.18343464249565E+00	0.36268378337836E+00
	5	0.52553240991633E+00	0.31370664587789E+00
	6	0.79666647741363E+00	0.22238103445337E+00
	7	0.96028985649754E+00	0.10122853629038E+00

CUADRATURAS DE GAUSS-LAGUERRE

$$\int_0^{+\infty} e^{-z} f(z)\,dz = \sum_{i=0}^{n} \omega_i f(z_i) + E_n, \qquad E_n = K_n\, f^{(2n+2)}(\xi), \quad \text{con} \begin{cases} K_n = \dfrac{((n+1)!)^2}{(2n+2)!}, \\ \xi \in [0,+\infty). \end{cases}$$

$$\int_a^{\infty} F(x)\,dx = \int_0^{\infty} e^{-z} f(z)\,dz, \qquad \text{donde} \begin{cases} x = a+z, \\ f(z) = e^{z} F(x). \end{cases}$$

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n	i	z_i	ω_i	$\omega_i e^{z_i}$
0	0	0.10000000000000E+01	0.10000000000000E+01	0.27182818284590E+01
1	0	0.58578643762690E+00	0.85355339059327E+00	0.15333260331194E+01
	1	0.34142135623731E+01	0.14644660940673E+00	0.44509573350546E+01
2	0	0.41577455678348E+00	0.71109300992917E+00	0.10776928592709E+01
	1	0.22942803602790E+01	0.27851773356924E+00	0.27621429619016E+01
	2	0.62899450829375E+01	0.10389256501586E-01	0.56010946254344E+01
3	0	0.32254768961939E+00	0.60315410434163E+00	0.83273912383789E+00
	1	0.17457611011583E+01	0.35741869243780E+00	0.20481024384543E+01
	2	0.45366202969211E+01	0.38887908515005E-01	0.36311463058215E+01
	3	0.93950709123011E+01	0.53929470556133E-03	0.64871450844077E+01
4	0	0.26356031971814E+00	0.52175561058281E+00	0.67909404220775E+00
	1	0.14134030591065E+01	0.39866681108318E+00	0.16384878736027E+01
	2	0.35964257710407E+01	0.75942449681708E-01	0.27694432423708E+01
	3	0.70858100058588E+01	0.36117586799221E-02	0.43156569009209E+01
	4	0.12640800844276E+02	0.23369972385776E-04	0.72191863543545E+01
5	0	0.22284660417926E+00	0.45896467394996E+00	0.57353550742274E+00
	1	0.11889321016726E+01	0.41700083077212E+00	0.13692525907123E+01
	2	0.29927363260593E+01	0.11337338207405E+00	0.22606845933827E+01
	3	0.57751435691045E+01	0.10399197453149E-01	0.33505245823555E+01
	4	0.98374674183826E+01	0.26101720281493E-03	0.48868268002108E+01
	5	0.15982873980602E+02	0.89854790642962E-06	0.78490159455958E+01
6	0	0.19304367656036E+00	0.40931895170127E+00	0.49647759753997E+00
	1	0.10266648953392E+01	0.42183127786172E+00	0.11776430608612E+01
	2	0.25678767449507E+01	0.14712634865751E+00	0.19182497816598E+01
	3	0.49003530845265E+01	0.20633514468717E-01	0.27718486362321E+01
	4	0.81821534445629E+01	0.10740101432807E-02	0.38412491224885E+01
	5	0.12734180291798E+02	0.15865464348564E-04	0.53806782079215E+01
	6	0.19395727862263E+02	0.31703154789956E-07	0.84054324868284E+01
7	0	0.17027963230510E+00	0.36918858934164E+00	0.43772341049291E+00
	1	0.90370177679938E+00	0.41878678081434E+00	0.10338693476656E+01
	2	0.22510866298661E+01	0.17579498663717E+00	0.16697097656588E+01
	3	0.42667001702877E+01	0.33343492261216E-01	0.23769247017586E+01
	4	0.70459054023935E+01	0.27945362352257E-02	0.32085409133479E+01
	5	0.10758516010181E+02	0.90765087733582E-04	0.42685755108251E+01
	6	0.15740678641278E+02	0.84857467162725E-06	0.58180833686719E+01
	7	0.22863131736889E+02	0.10480011748715E-08	0.89062262152922E+01

CUADRATURAS DE GAUSS-HERMITE

$$\int_{-\infty}^{+\infty} e^{-z^2} f(z)\,dz = \sum_{i=0}^{n} \omega_i\, f(z_i) + E_n, \qquad E_n = K_n\ f^{(2n+2)}(\xi), \quad \text{con} \begin{cases} K_n = \dfrac{\sqrt{\pi}\,(n+1)!}{2^{n+1}(2n+2)!}, \\ \xi \in (-\infty, +\infty). \end{cases}$$

$$\int_{-\infty}^{\infty} F(x)\,dx = \int_{-\infty}^{\infty} e^{-z^2} f(z)\,dz, \qquad \text{donde} \begin{cases} x = z, \\ f(z) = e^{z^2}\,F(x). \end{cases}$$

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n	i	z_i	ω_i	$\omega_i e^{z_i^2}$
0	0	0.00000000000000E+00	0.17724538509055E+01	0.17724538509055E+01
1	0	-0.70710678118655E+00	0.88622692545276E+00	0.14611411826611E+01
	1	0.70710678118655E+00	0.88622692545276E+00	0.14611411826611E+01
2	0	-0.12247448713916E+01	0.29540897515092E+00	0.13239311752136E+01
	1	0.00000000000000E+00	0.11816359006037E+01	0.11816359006037E+01
	2	0.12247448713916E+01	0.29540897515092E+00	0.13239311752136E+01
3	0	-0.16506801238858E+01	0.81312835447245E-01	0.12402258176958E+01
	1	-0.52464762327529E+00	0.80491409000551E+00	0.10599644828950E+01
	2	0.52464762327529E+00	0.80491409000551E+00	0.10599644828950E+01
	3	0.16506801238858E+01	0.81312835447245E-01	0.12402258176958E+01
4	0	-0.20201828704561E+01	0.19953242059046E-01	0.11814886255360E+01
	1	-0.95857246461382E+00	0.39361932315224E+00	0.98658099675143E+00
	2	0.00000000000000E+00	0.94530872048294E+00	0.94530872048294E+00
	3	0.95857246461382E+00	0.39361932315224E+00	0.98658099675143E+00
	4	0.20201828704561E+01	0.19953242059046E-01	0.11814886255360E+01
5	0	-0.23506049736745E+01	0.45300099055088E-02	0.11369083326745E+01
	1	-0.13358490740137E+01	0.15706732032286E+00	0.93558055763118E+00
	2	-0.43607741192762E+00	0.72462959522439E+00	0.87640133443623E+00
	3	0.43607741192762E+00	0.72462959522439E+00	0.87640133443623E+00
	4	0.13358490740137E+01	0.15706732032286E+00	0.93558055763118E+00
	5	0.23506049736745E+01	0.45300099055088E-02	0.11369083326745E+01
6	0	-0.26519613568352E+01	0.97178124509952E-03	0.11013307296103E+01
	1	-0.16735516287675E+01	0.54515582819127E-01	0.89718460022519E+00
	2	-0.81628788285896E+00	0.42560725261013E+00	0.82868730328364E+00
	3	0.00000000000000E+00	0.81026461755681E+00	0.81026461755681E+00
	4	0.81628788285896E+00	0.42560725261013E+00	0.82868730328364E+00
	5	0.16735516287675E+01	0.54515582819127E-01	0.89718460022519E+00
	6	0.26519613568352E+01	0.97178124509952E-03	0.11013307296103E+01
7	0	-0.29306374202572E+01	0.19960407221137E-03	0.10719301442480E+01
	1	-0.19816567566958E+01	0.17077983007413E-01	0.86675260656338E+00
	2	-0.11571937124468E+01	0.20780232581489E+00	0.79289004838640E+00
	3	-0.38118699020732E+00	0.66114701255824E+00	0.76454412865173E+00
	4	0.38118699020732E+00	0.66114701255824E+00	0.76454412865173E+00
	5	0.11571937124468E+01	0.20780232581489E+00	0.79289004838640E+00
	6	0.19816567566958E+01	0.17077983007413E-01	0.86675260656338E+00
	7	0.29306374202572E+01	0.19960407221137E-03	0.10719301442480E+01

CUADRATURAS DE GAUSS-TCHEBYSHEV (*)

$$\int_{-1}^{+1} \frac{1}{\sqrt{1-z^2}}\, f(z)\, dz = \sum_{i=0}^{n} \omega_i\, f(z_i) + E_n, \qquad E_n = K_n\; f^{(2n+2)}(\xi), \quad \text{con} \begin{cases} K_n = \dfrac{2\pi}{2^{2n+2}(2n+2)!}, \\ \xi \in [-1,+1]. \end{cases}$$

$$\int_a^b F(x)\, dx = \int_{-1}^{+1} \frac{1}{\sqrt{1-z^2}}\, f(z)\, dz, \qquad \text{donde} \begin{cases} x = \left(\frac{b+a}{2}\right) + \left(\frac{b-a}{2}\right) z, \\ f(z) = \left(\frac{b-a}{2}\right) \sqrt{1-z^2}\; F(x). \end{cases}$$

n	i	z_i	ω_i	$\omega_i \sqrt{1-z_i^2}$
0	0	0.00000000000000E+00	0.31415926535898E+01	0.31415926535898E+01
1	0	-0.70710678118655E+00	0.15707963267949E+01	0.11107207345396E+01
	1	0.70710678118655E+00	0.15707963267949E+01	0.11107207345396E+01
2	0	-0.86602540378444E+00	0.10471975511966E+01	0.52359877559830E+00
	1	0.00000000000000E+00	0.10471975511966E+01	0.10471975511966E+01
	2	0.86602540378444E+00	0.10471975511966E+01	0.52359877559830E+00
3	0	-0.92387953251129E+00	0.78539816339745E+00	0.30055886494217E+00
	1	-0.38268343236509E+00	0.78539816339745E+00	0.72561328803486E+00
	2	0.38268343236509E+00	0.78539816339745E+00	0.72561328803486E+00
	3	0.92387953251129E+00	0.78539816339745E+00	0.30055886494217E+00
4	0	-0.95105651629515E+00	0.62831853071796E+00	0.19416110387255E+00
	1	-0.58778525229247E+00	0.62831853071796E+00	0.50832036923153E+00
	2	0.00000000000000E+00	0.62831853071796E+00	0.62831853071796E+00
	3	0.58778525229247E+00	0.62831853071796E+00	0.50832036923153E+00
	4	0.95105651629515E+00	0.62831853071796E+00	0.19416110387255E+00
5	0	-0.96592582628907E+00	0.52359877559830E+00	0.13551733511720E+00
	1	-0.70710678118655E+00	0.52359877559830E+00	0.37024024484653E+00
	2	-0.25881904510252E+00	0.52359877559830E+00	0.50575757996373E+00
	3	0.25881904510252E+00	0.52359877559830E+00	0.50575757996373E+00
	4	0.70710678118655E+00	0.52359877559830E+00	0.37024024484653E+00
	5	0.96592582628907E+00	0.52359877559830E+00	0.13551733511720E+00
6	0	-0.97492791218182E+00	0.44879895051283E+00	0.99867161626728E-01
	1	-0.78183148246803E+00	0.44879895051283E+00	0.27982156872965E+00
	2	-0.43388373911756E+00	0.44879895051283E+00	0.40435388235934E+00
	3	0.00000000000000E+00	0.44879895051283E+00	0.44879895051283E+00
	4	0.43388373911756E+00	0.44879895051283E+00	0.40435388235934E+00
	5	0.78183148246803E+00	0.44879895051283E+00	0.27982156872965E+00
	6	0.97492791218182E+00	0.44879895051283E+00	0.99867161626728E-01
7	0	-0.98078528040323E+00	0.39269908169872E+00	0.76611790304042E-01
	1	-0.83146961230255E+00	0.39269908169872E+00	0.21817192032594E+00
	2	-0.55557023301960E+00	0.39269908169872E+00	0.32651735321160E+00
	3	-0.19509032201613E+00	0.39269908169872E+00	0.38515347895797E+00
	4	0.19509032201613E+00	0.39269908169872E+00	0.38515347895797E+00
	5	0.55557023301960E+00	0.39269908169872E+00	0.32651735321160E+00
	6	0.83146961230255E+00	0.39269908169873E+00	0.21817192032594E+00
	7	0.98078528040323E+00	0.39269908169872E+00	0.76611790304042E-01

(*) Para esta cuadratura: $z_i = \cos\left(\dfrac{2i+1}{2n+2}\,\pi\right), \quad \omega_i = \dfrac{\pi}{n+1}, \quad i = 0, \ldots, n.$

CUADRATURAS DE GAUSS-RADAU

$$\int_{-1}^{+1} f(z)\,dz = \sum_{i=0}^{n} \omega_i\, f(z_i) + E_n, \qquad E_n = K_n\; f^{(2n+1)}(\xi), \quad \text{con} \begin{cases} K_n = \dfrac{2^{2n+1}(n+1)(n!)^4}{((2n+1)!)^3}, \\ \xi \in [-1,+1]. \end{cases}$$

$$\int_a^b F(x)\,dx = \int_{-1}^{+1} f(z)\,dz, \qquad \text{donde} \begin{cases} x = \left(\dfrac{b+a}{2}\right) + \left(\dfrac{b-a}{2}\right)\, z, \\ f(z) = \left(\dfrac{b-a}{2}\right)\; F(x). \end{cases}$$

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n	i	z_i	ω_i
1	0	-0.10000000000000E+01	0.50000000000000E+00
	1	0.33333333333333E+00	0.15000000000000E+01
2	0	-0.10000000000000E+01	0.22222222222222E+00
	1	-0.28989794855664E+00	0.10249716523768E+01
	2	0.68989794855664E+00	0.75280612540093E+00
3	0	-0.10000000000000E+01	0.12500000000000E+00
	1	-0.57531892352169E+00	0.65768863996012E+00
	2	0.18106627111853E+00	0.77638693768634E+00
	3	0.82282408097459E+00	0.44092442235354E+00
4	0	-0.10000000000000E+01	0.80000000000000E-01
	1	-0.72048027131244E+00	0.44620780216714E+00
	2	-0.16718086473783E+00	0.62365304595148E+00
	3	0.44631397272375E+00	0.56271203029892E+00
	4	0.88579160777096E+00	0.28742712158245E+00
5	0	-0.10000000000000E+01	0.55555555555555E-01
	1	-0.80292982840235E+00	0.31964075322051E+00
	2	-0.39092854670727E+00	0.48538718846897E+00
	3	0.12405037950523E+00	0.52092678318957E+00
	4	0.60397316425278E+00	0.41690133431191E+00
	5	0.92038028589706E+00	0.20158838525348E+00
6	0	-0.10000000000000E+01	0.40816326530612E-01
	1	-0.85389134263948E+00	0.23922748922531E+00
	2	-0.53846772406011E+00	0.38094987364423E+00
	3	-0.11734303754310E+00	0.44710982901457E+00
	4	0.32603061943769E+00	0.42470377900596E+00
	5	0.70384280066303E+00	0.31820423146730E+00
	6	0.94136714568043E+00	0.14898847111202E+00
7	0	-0.10000000000000E+01	0.31250000000000E-01
	1	-0.88747487892616E+00	0.18535815480298E+00
	2	-0.63951861652622E+00	0.30413062064679E+00
	3	-0.29475056577366E+00	0.37651754538912E+00
	4	0.94307252661111E-01	0.39157216745249E+00
	5	0.46842035443082E+00	0.34701479563450E+00
	6	0.77064189367819E+00	0.24964790132987E+00
	7	0.95504122712257E+00	0.11450881474426E+00

CUADRATURAS DE GAUSS-LOBATTO

$$\int_{-1}^{+1} f(z)\,dz = \sum_{i=0}^{n} \omega_i\, f(z_i) + E_n, \qquad E_n = K_n\ f^{(2n)}(\xi), \quad \text{con} \begin{cases} K_n = \dfrac{-2^{2n+1}(n+1)(n!)^4}{n(2n+1)((2n)!)^3}, \\ \xi \in [-1,+1]. \end{cases}$$

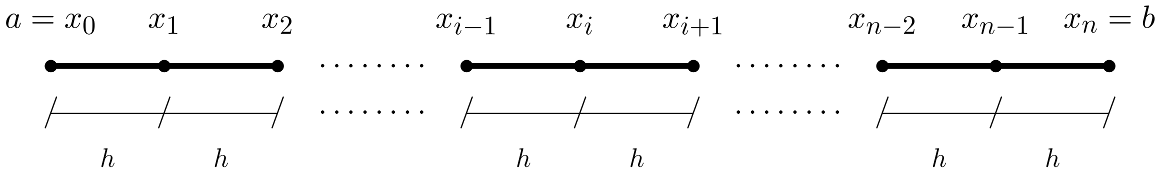
$$\int_a^b F(x)\,dx = \int_{-1}^{+1} f(z)\,dz, \qquad \text{donde} \begin{cases} x = \left(\dfrac{b+a}{2}\right) + \left(\dfrac{b-a}{2}\right)\ z, \\ f(z) = \left(\dfrac{b-a}{2}\right)\ F(x). \end{cases}$$

---

n	i	z_i	ω_i
2	0	-0.10000000000000E+01	0.33333333333333E+00
	1	0.00000000000000E+00	0.13333333333333E+01
	2	0.10000000000000E+01	0.33333333333333E+00
3	0	-0.10000000000000E+01	0.16666666666667E+00
	1	-0.44721359549996E+00	0.83333333333333E+00
	2	0.44721359549996E+00	0.83333333333333E+00
	3	0.10000000000000E+01	0.16666666666667E+00
4	0	-0.10000000000000E+01	0.10000000000000E+00
	1	-0.65465367070798E+00	0.54444444444444E+00
	2	0.00000000000000E+00	0.71111111111111E+00
	3	0.65465367070798E+00	0.54444444444444E+00
	4	0.10000000000000E+01	0.10000000000000E+00
5	0	-0.10000000000000E+01	0.66666666666666E-01
	1	-0.76505532392946E+00	0.37847495629785E+00
	2	-0.28523151648065E+00	0.55485837703549E+00
	3	0.28523151648065E+00	0.55485837703549E+00
	4	0.76505532392946E+00	0.37847495629785E+00
	5	0.10000000000000E+01	0.66666666666666E-01
6	0	-0.10000000000000E+01	0.47619047619048E-01
	1	-0.83022389627857E+00	0.27682604736157E+00
	2	-0.46884879347071E+00	0.43174538120986E+00
	3	0.00000000000000E+00	0.48761904761905E+00
	4	0.46884879347071E+00	0.43174538120986E+00
	5	0.83022389627857E+00	0.27682604736157E+00
	6	0.10000000000000E+01	0.47619047619048E-01
7	0	-0.10000000000000E+01	0.35714285714286E-01
	1	-0.87174014850961E+00	0.21070422714351E+00
	2	-0.59170018143314E+00	0.34112269248350E+00
	3	-0.20929921790248E+00	0.41245879465870E+00
	4	0.20929921790248E+00	0.41245879465870E+00
	5	0.59170018143314E+00	0.34112269248350E+00
	6	0.87174014850961E+00	0.21070422714351E+00
	7	0.10000000000000E+01	0.35714285714286E-01

CUADRATURAS COMPUESTAS

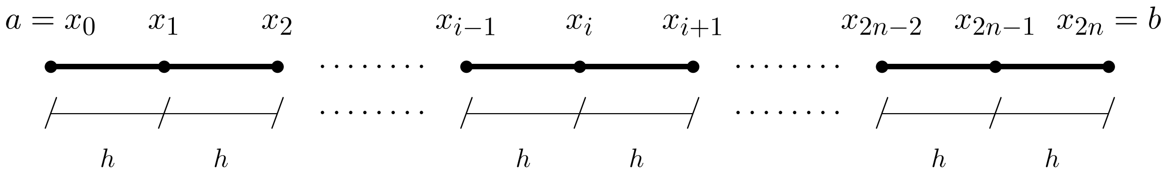
Cuadratura Compuesta del TRAPECIO



$$h = \frac{b-a}{n}, \qquad \xi \in [a,b], \qquad x_i = a + i\,h, \qquad f_i = f(x_i), \qquad i = 0, \ldots, n.$$

$$\begin{aligned} \int_a^b f(x)dx &= \frac{h}{2}\Big(f_0 + 2f_1 + 2f_2 + \ldots + 2f_{n-1} + f_n\Big) + E_n, \\ E_n &= -\frac{1}{12}\,\frac{(b-a)^3}{n^2}\,f^{(2)}(\xi). \quad (*) \end{aligned}$$

Cuadratura Compuesta de SIMPSON



$$h = \frac{b-a}{2n}, \qquad \xi \in [a,b], \qquad x_i = a + i\,h, \qquad f_i = f(x_i), \qquad i = 0, \ldots, 2n.$$

$$\begin{aligned} \int_a^b f(x)dx &= \frac{h}{3}\Big(f_0 + 4f_1 + 2f_2 + 4f_3 + \ldots + 2f_{2n-2} + 4f_{2n-1} + f_{2n}\Big) + E_{2n}, \\ E_{2n} &= -\frac{1}{180}\,\frac{(b-a)^5}{(2n)^4}\,f^{(4)}(\xi). \quad (**) \end{aligned}$$

(*) Cuando $n \to \infty$, $\quad f^{(2)}(\xi) \to \dfrac{f'(b) - f'(a)}{b-a} \implies E_n = \mathcal{O}\left(h^2\right).$

(**) Cuando $n \to \infty$, $\quad f^{(4)}(\xi) \to \dfrac{f^{(3)}(b) - f^{(3)}(a)}{b-a} \implies E_{2n} = \mathcal{O}\left(h^4\right).$

EXTRAPOLACIÓN DE RICHARDSON

Sean:

$I_1 \equiv$ integral calculada con n_1 subintervalos;
$I_2 \equiv$ integral calculada con n_2 subintervalos;
$I_R \equiv$ extrapolación de Richardson (valor mejorado de la integral).

Cuadratura Compuesta del TRAPECIO

$$I_R = \frac{I_2 (n_2/n_1)^2 - I_1}{(n_2/n_1)^2 - 1}; \qquad n_2 = 2\, n_1 \longrightarrow I_R = \frac{4I_2 - I_1}{3}.$$

Cuadratura Compuesta de SIMPSON

$$I_R = \frac{I_2 (n_2/n_1)^4 - I_1}{(n_2/n_1)^4 - 1}; \qquad n_2 = 2\, n_1 \longrightarrow I_R = \frac{16I_2 - I_1}{15}.$$

MÉTODO DE INTEGRACIÓN DE ROMBERG

Metodo de Romberg $\Longrightarrow \begin{cases} \text{realizar sucesivas extrapolaciones de Richarson} \\ \text{sobre los resultados de la fórmula del trapecio compuesta.} \end{cases}$

Los valores obtenidos se tabulan en la forma:

más extrapolaciones $\longrightarrow$

más puntos $\downarrow$

$$
\begin{array}{ccccccccccc}
T_{0,1} & \to & T_{0,2} & \to & T_{0,3} & \to & T_{0,4} & \to & \dots & \to & T_{0,N+1} \\
 & \nearrow & & \nearrow & & \nearrow & & \nearrow & & \nearrow & \\
T_{1,1} & \to & T_{1,2} & \to & T_{1,3} & \to & \dots & \to & \cdot^{\cdot^{\cdot}} & & \\
 & \nearrow & & \nearrow & & \nearrow & & \nearrow & & & \\
T_{2,1} & \to & T_{2,2} & \to & \dots & \to & \cdot^{\cdot^{\cdot}} & & & & \\
 & \nearrow & & \nearrow & & \nearrow & & & & & \\
T_{3,1} & \to & \dots & \to & \cdot^{\cdot^{\cdot}} & & & & & & \\
 & \nearrow & & \nearrow & & & & & & & \\
\vdots & & \cdot^{\cdot^{\cdot}} & & & & & & & & \\
 & \nearrow & & & & & & & & & \\
T_{N,1} & & & & & & & & & &
\end{array}
$$

donde $\begin{cases} j = 1 \;\Rightarrow\; T_{i,1} \equiv \text{fórmula del trapecio compuesta con } 2^i \text{ intervalos.} \\ j > 1 \;\Rightarrow\; T_{i,j} \equiv \text{extrapolación de Richardson entre } T_{i,j-1} \text{ y } T_{i+1,j-1}. \end{cases}$

Algoritmo:

$$T_{0,1} = \frac{b-a}{2}\Big(f(a) + f(b)\Big).$$

$$T_{i,1} = \frac{1}{2}\left(T_{i-1,1} + \frac{b-a}{2^{i-1}} \sum_{k=1,2^i-1,2} f(a + \frac{b-a}{2^i}k)\right); \qquad i = 1, \dots, N.$$

$$T_{i,j} = \frac{4^{j-1}T_{i+1,j-1} - T_{i,j-1}}{4^{j-1} - 1}; \qquad i = 0, \dots, N-j+1; \quad j = 2, \dots, N+1.$$

Se puede demostrar que:

1) $\forall j,\ \lim_{i\to\infty} T_{i,j} = \int_a^b f(x)dx; \qquad \forall i,\ \lim_{j\to\infty} T_{i,j} = \int_a^b f(x)dx.$ (*)

2) $\int_a^b f(x)dx = T_{i,j} + E_{i,j}$, con $E_{i,j} = \dfrac{K(a,b,j)}{4^{j\cdot i}} f^{(2j)}(\xi),\ \xi \in [a,b].$ (**)

(*) Todas las filas y todas las columnas convergen al valor exacto de la integral.
(**) Los valores más precisos se esperan en la parte media del lado inferior del triángulo.

Comparación: Newton-Cotes / Cuadraturas Compuestas

Resultados obtenidos mediante distintas cuadraturas y un número creciente de puntos de integración para el cálculo de la integral

$$I = \int_{-4}^{+4} \frac{1}{1+x^2}\, dx.$$

Cuadraturas Cerradas de NEWTON-COTES

Interv.	Val. Exac.	Puntos	Val. Aprox	Err. Rel.(%)
1	2.6516353273	2	0.4705882353	82.252905200
2	2.6516353273	3	5.4901960784	-107.049439334
3	2.6516353273	4	2.2776470588	14.104061168
4	2.6516353273	5	2.2776470588	14.104061168
5	2.6516353273	6	2.3722292496	10.537123067
6	2.6516353273	7	3.3287981275	-25.537553869
7	2.6516353273	8	2.7997007825	-5.583929797
8	2.6516353273	9	1.9410943044	26.796332649
9	2.6516353273	10	2.4308411566	8.326717042
10	2.6516353273	11	3.5955604002	-35.597846473

Fórmulas Compuestas del Trapecio y Simpson 1/3

		Fórmula Compuesta del Trapecio			Fórmula Compuesta Simpson 1/3		
Interv.	Val. Exac.	Puntos	Val. Aprox	Err. Rel.(%)	Puntos	Val. Aprox	Err. Rel.(%)
1	2.6516353273	2	0.4705882353	82.252905200	3	5.4901960784	-107.049439334
2	2.6516353273	3	4.2352941176	-59.723853201	5	2.4784313725	6.531967386
3	2.6516353273	4	2.0768627451	21.676154949	7	2.9084215239	-9.684069068
4	2.6516353273	5	2.9176470588	-10.031987760	9	2.5725490196	2.982548426
5	2.6516353273	6	2.5187099403	5.012958820	11	2.6952859224	-1.646176402
6	2.6516353273	7	2.7005318292	-1.844013064	13	2.6332910535	0.691809828
7	2.6516353273	8	2.6200579018	1.190866075	15	2.6602997673	-0.326758355
8	2.6516353273	9	2.6588235294	-0.271085620	17	2.6477345635	0.147107854
9	2.6516353273	10	2.6426739520	0.337956553	19	2.6534158938	-0.067149749
10	2.6516353273	11	2.6511419269	0.018607403	21	2.6508184459	0.030806705
11	2.6516353273	12	2.6480963324	0.133464618	23	2.6520029475	-0.013863904
12	2.6516353273	13	2.6501012474	0.057854105	25	2.6514640295	0.006460083
13	2.6516353273	14	2.6496637693	0.074352533	27	2.6517107066	-0.002842745
14	2.6516353273	15	2.6502393009	0.052647752	29	2.6515989345	0.001372466
15	2.6516353273	16	2.6502787921	0.051158439	31	2.6516503898	-0.000568044
16	2.6516353273	17	2.6505068050	0.042559485	33	2.6516272830	0.000303374
17	2.6516353273	18	2.6506059693	0.038819742	35	2.6516380757	-0.000103647
18	2.6516353273	19	2.6507304083	0.034126827	37	2.6516333457	0.000074734
19	2.6516353273	20	2.6508168216	0.030867960	39	2.6516356461	-0.000012020
20	2.6516353273	21	2.6508993161	0.027756879	41	2.6516347072	0.000023386
25	2.6516353273	26	2.6511633755	0.017798521	51	2.6516352085	0.000004480
30	2.6516353273	31	2.6513074904	0.012363577	61	2.6516352668	0.000002281
40	2.6516353273	41	2.6514508595	0.006956759	81	2.6516353082	0.000000721
50	2.6516353273	51	2.6515172503	0.004452990	101	2.6516353195	0.000000296
100	2.6516353273	101	2.6516058022	0.001113469	201	2.6516353268	0.000000018
200	2.6516353273	201	2.6516279457	0.000278381	401	2.6516353273	0.000000001
400	2.6516353273	401	2.6516334819	0.000069596	801	2.6516353273	0.000000000
800	2.6516353273	801	2.6516348660	0.000017399	1601	2.6516353273	0.000000000