Ejercicios propuestos en la clase del 07/10/2024

Obtener la solución de equilibrio, es decir la solución correspondiente al estado estacionario, de los siguientes problemas:

1.–

$$\begin{aligned} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + Q_0, \quad 0 < x < L, \quad t > 0 \\ u(x,0) = f(x), \quad 0 \le x \le L \\ \frac{\partial u(0,t)}{\partial x} = 0, \quad \frac{\partial u(L,t)}{\partial x} = 0, \quad t \ge 0 \end{aligned}$$

siendo Q_0 una constante real no nula.

Solución 1. No existe solución de equilibrio para este caso.

2.–

$$\begin{aligned} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + Q_0, \quad 0 < x < L, \quad t > 0 \\ u(x,0) = f(x), \quad 0 \le x \le L \\ u(0,t) = 0, \quad u(L,t) = 0, \quad t \ge 0 \end{aligned}$$

siendo Q_0 una constante real no nula.

Solución 2. La solución de equilibrio es $u_E(x) = \dfrac{Q_0\, x(L-x)}{2}$.

3.–

$$\begin{aligned} \frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \sin(x), \quad 0 < x < \pi, \quad t > 0 \\ u(x,0) = 1 + \sin(x), \quad 0 \le x \le \pi \\ u(0,t) = 1, \quad \frac{\partial u(\pi,t)}{\partial x} = 2, \quad t \ge 0 \end{aligned}$$

Solución 3. La solución de equilibrio es $u_E(x) = 1 + 3x + \sin(x)$.

4.–

$$\begin{aligned} \frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left((1+x)\frac{\partial u}{\partial x}\right) + 2(1+x), \quad 0 < x < L, \quad t > 0 \\ u(x,0) = f(x), \quad 0 \le x \le L \\ \frac{\partial u(0,t)}{\partial x} = 0, \quad u(L,t) = 0, \quad t \ge 0 \end{aligned}$$

Solución 4. La solución de equilibrio es $u_E(x) = \ln\left(\dfrac{x+1}{L+1}\right) + \dfrac{(L+1)^2 - (x+1)^2}{2}$.

5.–

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(e^{-x}\frac{\partial u}{\partial x}\right), \quad 0 < x < 10, \quad t > 0$$
$$u(x,0) = f(x), \quad 0 \le x \le 10$$
$$u(0,t) - \frac{\partial u(0,t)}{\partial x} = 320, \quad t \ge 0$$
$$u(10,t) + \frac{\partial u(10,t)}{\partial x} = 320, \quad t \ge 0$$

Solución 5. La solución de equilibrio es $u_E(x) = 320$.

6.–

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(e^{-x}\frac{\partial u}{\partial x}\right), \quad 0 < x < 10, \quad t > 0$$
$$u(x,0) = f(x), \quad 0 \le x \le 10$$
$$u(0,t) - \frac{\partial u(0,t)}{\partial x} = 320, \quad t \ge 0$$
$$u(10,t) + \frac{\partial u(10,t)}{\partial x} = 300, \quad t \ge 0$$

Solución 6. La solución de equilibrio es $u_E(x) = 320 - 10e^{(x-10)}$.
