

Ejercicios propuestos en la clase del 10/10/2023

Obtener la solución de equilibrio, es decir la solución correspondiente al estado estacionario, de los siguientes problemas:

1.-

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} + Q_0, & 0 < x < L, & \quad t > 0 \\ u(x, 0) &= f(x), & 0 \leq x \leq L \\ \frac{\partial u(0, t)}{\partial x} &= 0, & \frac{\partial u(L, t)}{\partial x} &= 0, \quad t \geq 0\end{aligned}$$

siendo Q_0 una constante real no nula.

Solución 1. No existe solución de equilibrio para este caso.

2.-

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} + Q_0, & 0 < x < L, & \quad t > 0 \\ u(x, 0) &= f(x), & 0 \leq x \leq L \\ u(0, t) &= 0, & u(L, t) &= 0, \quad t \geq 0\end{aligned}$$

siendo Q_0 una constante real no nula.

Solución 2. La solución de equilibrio es $u_E(x) = \frac{Q_0 x(L-x)}{2}$.

3.-

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} + \sin(x), & 0 < x < \pi, & \quad t > 0 \\ u(x, 0) &= 1 + \sin(x), & 0 \leq x \leq \pi \\ u(0, t) &= 1, & \frac{\partial u(\pi, t)}{\partial x} &= 2, \quad t \geq 0\end{aligned}$$

Solución 3. La solución de equilibrio es $u_E(x) = 1 + 3x + \sin(x)$.

4.-

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial}{\partial x} \left((1+x) \frac{\partial u}{\partial x} \right) + 2(1+x), & 0 < x < L, & \quad t > 0 \\ u(x, 0) &= f(x), & 0 \leq x \leq L \\ \frac{\partial u(0, t)}{\partial x} &= 0, & u(L, t) &= 0, \quad t \geq 0\end{aligned}$$

Solución 4. La solución de equilibrio es $u_E(x) = \ln\left(\frac{x+1}{L+1}\right) + \frac{(L+1)^2 - (x+1)^2}{2}$.

5.−

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial}{\partial x} \left(e^{-x} \frac{\partial u}{\partial x} \right), & 0 < x < 10, & \quad t > 0 \\ u(x, 0) &= f(x), & 0 \leq x \leq 10 \\ u(0, t) - \frac{\partial u(0, t)}{\partial x} &= 320, & t \geq 0 \\ u(10, t) + \frac{\partial u(10, t)}{\partial x} &= 320, & t \geq 0\end{aligned}$$

Solución 5. La solución de equilibrio es $u_E(x) = 320$.

6.−

$$\begin{aligned}\frac{\partial u}{\partial t} &= \frac{\partial}{\partial x} \left(e^{-x} \frac{\partial u}{\partial x} \right), & 0 < x < 10, & \quad t > 0 \\ u(x, 0) &= f(x), & 0 \leq x \leq 10 \\ u(0, t) - \frac{\partial u(0, t)}{\partial x} &= 320, & t \geq 0 \\ u(10, t) + \frac{\partial u(10, t)}{\partial x} &= 300, & t \geq 0\end{aligned}$$

Solución 6. La solución de equilibrio es $u_E(x) = 320 - 10e^{(x-10)}$.
