MÉTODOS DE INTERVALO SIMPLE

1.1. Métodos basados en la aproximación de la derivada

1.1.1. Método de Euler

$$y_{i+1} = y_i + h\varphi(x_i, y_i) \qquad ; \tau(h)$$

1.1.2 Método de Diferencias Centradas

$$y_{i+1} = y_{i-1} + 2h\varphi(x_i, y_i) \qquad ; \tau(h^2)$$

1.2. Métodos basados en desarrollos en serie

1.2.1. Método del Desarrollo en Serie de Segundo Orden

$$y_{i+1} = y_i + h\varphi(x_i, y_i) + \frac{h^2}{2}\left(\varphi'_x(x_i, y_i) + \varphi'_y(x_i, y_i)\varphi(x_i, y_i)\right) \qquad ; \tau(h^2)$$

1.3. Métodos de Runge-Kutta

1.3.1. Métodos de Runge-Kutta de Segundo Orden

$$\begin{aligned} y_{i+1} = y_i + h\Phi(x_i, y_i) \\ \Phi(x, y) &= w_0 k_0 + w_1 k_1 \\ k_0 &= \varphi(x, y) \\ k_1 &= \varphi(x + \theta_1 h, y + (w_{10} k_0)h) \end{aligned}$$

$$w_0 + w_1 = 1; \qquad w_1\theta_1 = \frac{1}{2}; \qquad w_1 w_{10} = \frac{1}{2}$$

1.3.1.1. Método de Euler Modificado

$$w_0 = 0; \qquad w_1 = 1; \qquad \theta_1 = \frac{1}{2}; \qquad w_{10} = \frac{1}{2} \qquad ; \tau(h^2)$$

1.3.1.2. Método de Heun

$$w_0 = \frac{1}{2}; \qquad w_1 = \frac{1}{2}; \qquad \theta_1 = 1; \qquad w_{10} = 1 \qquad ; \tau(h^2)$$

1.3.2. Métodos de Runge-Kutta de Tercer Orden

$$
\begin{aligned}
y_{i+1} = y_i + h\Phi(x_i, y_i)& \\
\Phi(x, y) =& w_0 k_0 + w_1 k_1 + w_2 k_2 \\
k_0 =& \varphi(x, y) \\
k_1 =& \varphi(x + \theta_1 h, y + (w_{10} k_0) h) \\
k_2 =& \varphi(x + \theta_2 h, y + (w_{20} k_0 + w_{21} k_1) h)
\end{aligned}
$$

1.3.2.1. Método de Heun de Tercer Orden

$$
\begin{matrix}
w_0 = \frac{1}{4}; & w_1 = 0; & w_2 = \frac{3}{4} \\
\theta_1 = \frac{1}{3}; & \theta_2 = \frac{2}{3}; & \\
w_{10} = \frac{1}{3}; & w_{20} = 0; & w_{21} = \frac{2}{3}
\end{matrix} \qquad ; \tau(h^3)
$$

1.3.3. Métodos de Runge-Kutta de Cuarto Orden

$$
\begin{aligned}
y_{i+1} = y_i + h\Phi(x_i, y_i)& \\
\Phi(x, y) =& w_0 k_0 + w_1 k_1 + w_2 k_2 + w_3 k_3 \\
k_0 =& \varphi(x, y) \\
k_1 =& \varphi(x + \theta_1 h, y + (w_{10} k_0) h) \\
k_2 =& \varphi(x + \theta_2 h, y + (w_{20} k_0 + w_{21} k_1) h) \\
k_3 =& \varphi(x + \theta_3 h, y + (w_{30} k_0 + w_{31} k_1 + w_{32} k_2) h)
\end{aligned}
$$

1.3.3.1. Método de Kutta de Cuarto Orden

$$
\begin{matrix}
w_0 = \frac{1}{6}; & w_1 = \frac{1}{3}; & w_2 = \frac{1}{3}; & w_3 = \frac{1}{6} \\
\theta_1 = \frac{1}{2}; & \theta_2 = \frac{1}{2}; & \theta_3 = 1 & \\
w_{10} = \frac{1}{2}; & w_{20} = 0; & w_{21} = \frac{1}{2} & \\
w_{30} = 0; & w_{31} = 0; & w_{32} = 1 &
\end{matrix} \qquad ; \tau(h^4)
$$

1.3.3.1. Método de Gill

$$
\begin{matrix}
w_0 = \frac{1}{6}; & w_1 = \frac{2}{6}(1 - \frac{1}{\sqrt{2}}); & w_2 = \frac{2}{6}(1 + \frac{1}{\sqrt{2}}); & w_3 = \frac{1}{6} \\
\theta_1 = \frac{1}{2}; & \theta_2 = \frac{1}{2}; & \theta_3 = 1 & \\
w_{10} = \frac{1}{2}; & w_{20} = (-\frac{1}{2} + \frac{1}{\sqrt{2}}); & w_{21} = (1 - \frac{1}{\sqrt{2}}) & \\
w_{30} = 0; & w_{31} = -\frac{1}{\sqrt{2}}; & w_{32} = (1 + \frac{1}{\sqrt{2}}) &
\end{matrix} \qquad ; \tau(h^4)
$$