1.a) $$I=\int_{y=0}^{y=6}\left[\underbrace{\int_{x=0}^{x=12-2y}\underbrace{\left(\int_{z=0}^{z=4-\frac{2y}{3}-x/3} x\,dz\right)}_{g(y,x)=x\left(4-\frac{2y}{3}-\frac{x}{3}\right)}dx}_{h(y)=4/9(6-y)^3}\right]dy = 144$$

1.b) $$I=\int_{x=-1}^{x=1}\left[\underbrace{\int_{y=2}^{y=3}\underbrace{\left(\int_{z=0}^{z=1}(xy+yz)\,dz\right)}_{g(x,y)=y(x+1/2)}dy}_{h(x)=5/2(x+1/2)}\right]dx = 5/2$$

1.c) $$I=\int_{z=-1}^{z=1}\left[\underbrace{\int_{y=0}^{y=1}\underbrace{\left(\int_{x=y^2}^{x=1-z}dx\right)}_{g(z,y)=(1-z)-y^2}dy}_{h(z)=2/3-z}\right]dz = 4/3$$

1.d) $$I=\int_{\theta=0}^{\theta=\pi/2}\left[\underbrace{\int_{z=0}^{z=4}\underbrace{\left(\int_{\rho=0}^{\rho=\sqrt{16-z^2}}\sqrt{16-\rho^2}\;z\rho\,d\rho\right)}_{g(\theta,z)=-1/3(z^4-64z)}dz}_{h(\theta)=1/5\cdot 2^9}\right]d\theta=\frac{256}{5}\pi$$

1.e) $$I=\int_{\theta=0}^{\theta=\pi/2}\left[\underbrace{\int_{\rho=0}^{\rho=a}\underbrace{\left(\int_{z=0}^{z=\sqrt{a^2-\rho^2}}z\,\rho\,dz\right)}_{g(\theta,\rho)=1/2(a^2\rho-\rho^3)}d\rho}_{h(\theta)=a^4/8}\right]d\theta = a^4/16\,\pi$$

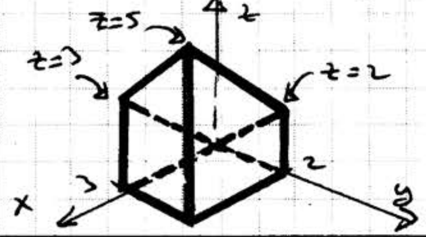
1.f) $$I=\int_{\theta=0}^{\theta=2\pi}\left[\underbrace{\int_{\varphi=0}^{\varphi=\pi}\underbrace{\left(\int_{r=0}^{r=5}r^4\,\text{sen}\,\varphi\,dr\right)}_{g(\theta,\varphi)=5^4\,\text{sen}\,\varphi}d\varphi}_{h(\theta)=2.5^4}\right]d\theta=2500\,\pi$$

1.g) $$I=\int_{\theta=0}^{\theta=2\pi}\left[\underbrace{\int_{\varphi=0}^{\varphi=\pi/2}\underbrace{\left(\int_{r=0}^{r=2a\cos\varphi}r^3\cos\varphi\,\text{sen}\,\varphi\,dr\right)}_{g(\theta,\varphi)=4a^4\cos^5\varphi\,\text{sen}\,\varphi}d\varphi}_{h(\theta)=\frac{37}{96}a^4}\right]d\theta=\frac{37}{48}a^4\cdot\pi$$

2.a)

$$I=\int_{x=0}^{x=3}\Bigg[\underbrace{\int_{y=0}^{y=2}\Bigg(\underbrace{\int_{z=0}^{z=x+y} 2xy\;dz}_{g(x,y)}\Bigg)dy}_{h(x)}\Bigg]dx$$

$g(x,y)=2xy(x+y)$; $h(x)=2x(2x+8/3)$; $I=60$

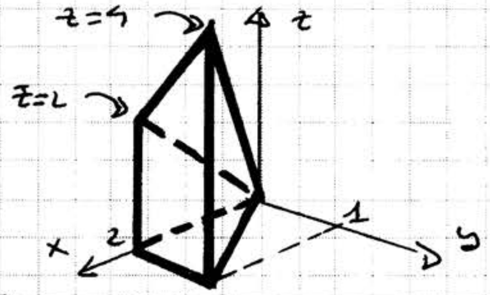


$$I=\iiint_V 2xy\;dV$$

2.b)

$$I=\int_{y=0}^{y=1}\Bigg[\underbrace{\int_{x=2y}^{x=2}\Bigg(\underbrace{\int_{z=0}^{z=x+2y}(x-2z)\,dz}_{g(y,x)}\Bigg)dx}_{h(y)}\Bigg]dy$$

$g(y,x)=-(2yx+4y^2)$; $h(y)=-4y-8y^2+12y^3$; $I=-5/3$

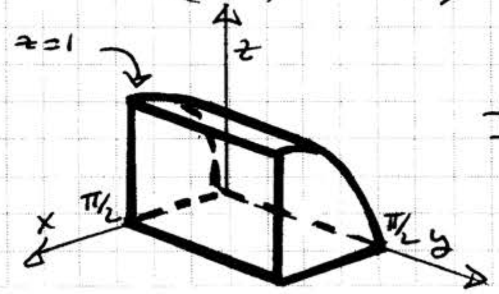


$$I=\iiint_V (x-2z)\,dV$$

2.c)

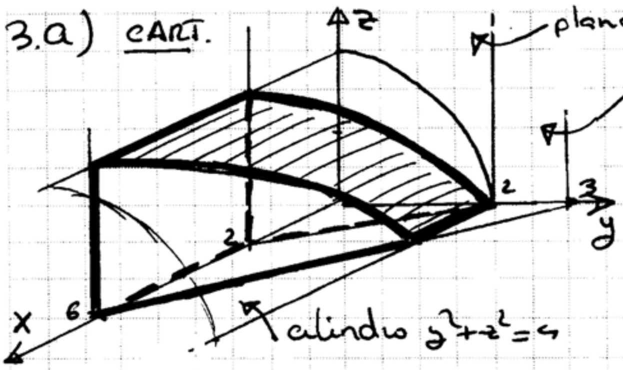
$$I=\int_{y=0}^{y=\pi/2}\Bigg[\underbrace{\int_{x=0}^{x=\pi/2}\Bigg(\underbrace{\int_{z=0}^{z=\operatorname{sen}(x)}(y-x)\,dz}_{g(y,x)}\Bigg)dx}_{h(y)}\Bigg]dy$$

$g(y,x)=(y-x)\operatorname{sen}(x)$; $h(y)=-1+y$; $I=-\pi/2+\pi^2/8$



$$I=\iiint_V (y-x)\,dV$$

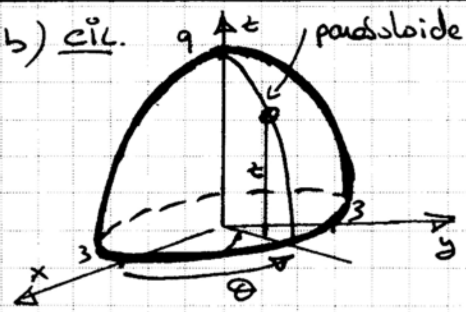
3.a) CART.



$$I=\int_{y=0}^{y=2}\overbrace{\left[\int_{x=2-y}^{x=6-2y}\underbrace{\left(\int_{z=0}^{z=\sqrt{4-y^2}} z\,dz\right)}_{g(y,x)}dx\right]}^{h(y)}dy$$

$g(y,x)=2-y^2/2$; $h(y)=8-2y-2y^2+y^3/2$; $I=26/3$

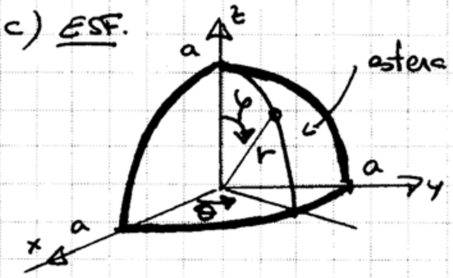
3.b) CIL.



$$I=\int_{\theta=0}^{\theta=2\pi}\overbrace{\left[\int_{\rho=0}^{\rho=3}\underbrace{\left(\int_{z=0}^{z=9-\rho^2}\rho^2|\rho|\,dz\right)}_{g(\theta,\rho)}d\rho\right]}^{h(\theta)}d\theta$$

$g(\theta,\rho)=9\rho^3-\rho^5$; $h(\theta)=3^5/4$; $I=\frac{243}{2}\cdot\pi$

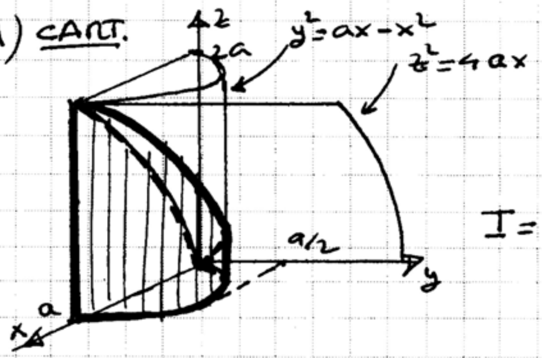
3.c) ESF.



$$I=\int_{\theta=0}^{\theta=\pi/2}\overbrace{\left[\int_{\varphi=0}^{\varphi=\pi/2}\underbrace{\left(\int_{r=0}^{r=a} r\cos\varphi\,|r^2\operatorname{sen}\varphi|\,dr\right)}_{g(\theta,\varphi)}d\varphi\right]}^{h(\theta)}d\theta$$

$g(\theta,\varphi)=a^4/4\,\cos\varphi\,\operatorname{sen}\varphi$; $h(\theta)=a^4/8$; $I=\pi a^4/16$

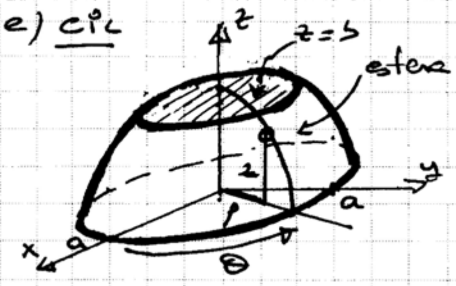
3.d) CART.



$$I=\int_{x=0}^{x=a}\overbrace{\left[\int_{y=0}^{y=\sqrt{ax-x^2}}\underbrace{\left(\int_{z=0}^{z=\sqrt{4ax}} x^2yz^3\,dz\right)}_{g(x,y)}dy\right]}^{h(x)}dx$$

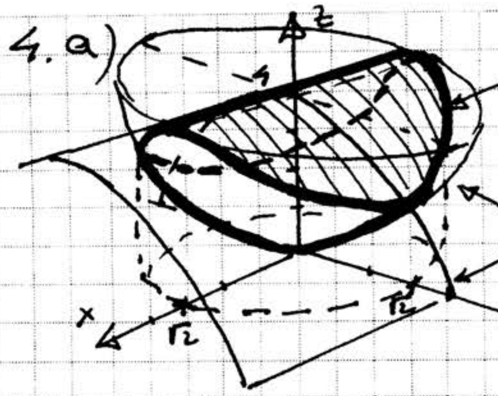
$g(x,y)=4a^2x^4y$; $h(x)=2a^3x^5-2a^2x^6$; $I=a^9/21$

3.e) CIL



$$I=\int_{\theta=0}^{\theta=2\pi}\overbrace{\left[\int_{z=0}^{z=b}\underbrace{\left(\int_{\rho=0}^{\rho=\sqrt{a^2-z^2}}\frac{1}{\sqrt{\rho^2+z^2}}|\rho|\,d\rho\right)}_{g(\theta,z)}dz\right]}^{h(\theta)}d\theta$$

$g(\theta,z)=a-z$; $h(\theta)=ab-b^2/2$; $I=\pi b(2a-b)$

4. a) 

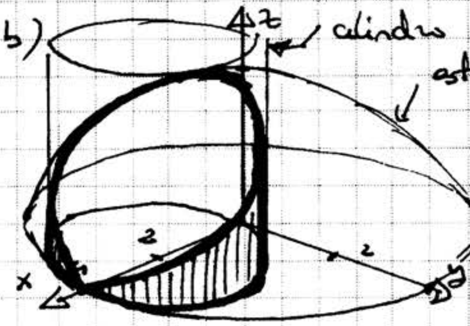
intersección $\left\{\begin{matrix} z=2x^2+y^2 \\ z=4-y^2 \end{matrix}\right\} \Rightarrow x^2+y^2=2$ CART.

paraboloide $z=2x^2+y^2$

cilindro $z=4-y^2$

$$I=\int_{y=-\sqrt{2}}^{y=+\sqrt{2}}\underbrace{\left[\int_{x=-\sqrt{2-y^2}}^{x=+\sqrt{2-y^2}}\left(\underbrace{\int_{z=2x^2+y^2}^{z=4-y^2}dz}_{g(y,x)}\right)dx\right]}_{h(y)}dy$$

$g(y,x)=4-2x^2-2y^2$; $h(y)=8/3\,(2-y^2)^{3/2}$; $I=4\pi$

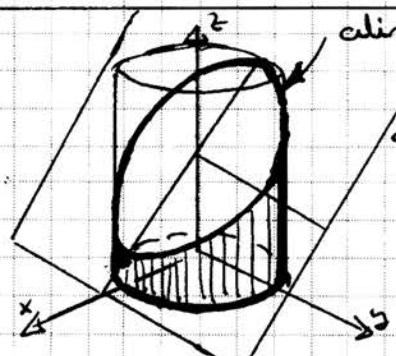
4. b) 

cilindro $(x-2)^2+y^2=4$ CIL.

esfera $x^2+y^2+z^2=16$

$$I=2\cdot\int_{\theta=0}^{\theta=\pi/2}\underbrace{\left[\int_{\rho=0}^{\rho=4\cos\theta}\left(\underbrace{\int_{z=0}^{z=\sqrt{16-\rho^2}}|\rho|\,dz}_{g(\theta,\rho)}\right)d\rho\right]}_{h(\theta)}d\theta$$

$g(\theta,\rho)=\rho\sqrt{16-\rho^2}$; $h(\theta)=\frac{-64}{3}\left(|\operatorname{sen}\theta|^3-1\right)$; $I=\frac{64}{9}(3\pi-4)$

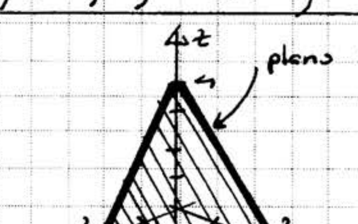
4. c) 

cilindro $x^2+y^2=9$ CIL.

plano $x+z=4$

$$I=2\int_{\theta=0}^{\theta=\pi}\underbrace{\left[\int_{\rho=0}^{\rho=3}\left[\underbrace{\int_{z=0}^{z=4-\rho\cos\theta}|\rho|\,dz}_{g(\theta,\rho)}\right]d\rho\right)}_{h(\theta)}d\theta$$

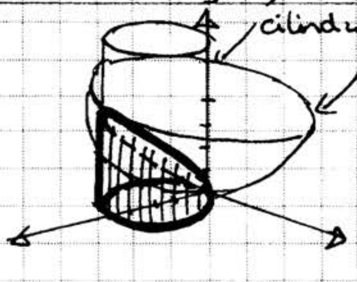
$g(\theta,\rho)=4\rho-\rho^2\cos\theta$; $h(\theta)=18-9\cos\theta$; $I=36\pi$

4. d) 

plano $6x+4y+3z=12$; $z=0\Rightarrow y=3-3/2\cdot x$ CART.

$$I=\int_{x=0}^{x=2}\underbrace{\left[\int_{y=0}^{y=3-3/2x}\left(\underbrace{\int_{z=0}^{z=4-2x-4/3y}dz}_{g(x,y)}\right)dy\right]}_{h(x)}dx$$

$g(x,y)=4-2x-4/3\,y$; $h(x)=6-6x+3x^2/2$; $I=4$

4. e) 

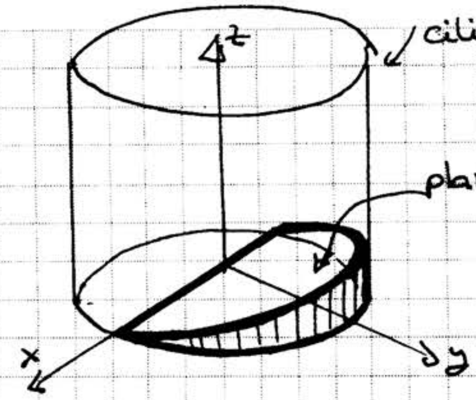
cilindro $x^2+y^2=4x$ CIL.

paraboloide $x^2+y^2=4z$

$$I=2\int_{\theta=0}^{\theta=\pi/2}\underbrace{\left[\int_{\rho=0}^{\rho=4\cos\theta}\left(\underbrace{\int_{z=0}^{z=1/4\rho^2}|\rho|\,dz}_{g(\theta,\rho)}\right)d\rho\right]}_{h(\theta)}d\theta$$

$g(\theta,\rho)=1/4\,\rho^3$; $h(\theta)=16\cos^4\theta$; $I=6\pi$

4.f) **CIL**



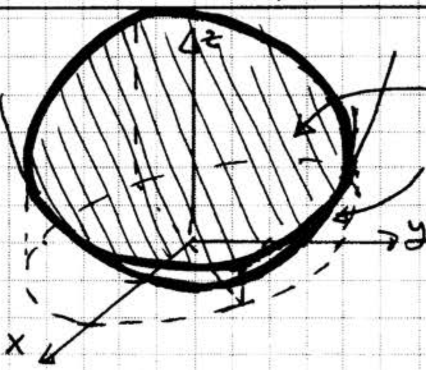
cilindro $x^2+y^2=16$

plano $z=y/2$

$$I = 2\int_{\theta=0}^{\theta=\pi/2}\underbrace{\left[\int_{\rho=0}^{\rho=4}\left(\underbrace{\int_{z=0}^{z=\rho\,\mathrm{sen}\,\theta/2} |\rho|\,dz}_{g(\theta,\rho)}\right)d\rho\right]}_{h(\theta)}d\theta$$

$g(\theta,\rho) = \frac{1}{2}\rho^2\,\mathrm{sen}\,\theta$; $h(\theta) = \frac{32}{3}\,\mathrm{sen}\,\theta$; $I = 64/3$

4.g) **CIL**



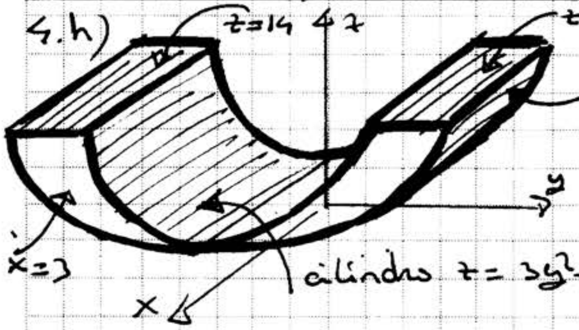
plano $z = 6-2x-2y$

paraboloide $z=(x-1)^2+(y-1)^2$ $\Rightarrow$ intersección: $x^2+y^2=4$

$$I = \int_{\theta=0}^{\theta=2\pi}\underbrace{\left[\int_{\rho=0}^{\rho=2}\left(\underbrace{\int_{z=(\rho\cos\theta-1)^2+(\rho\,\mathrm{sen}\,\theta-1)^2}^{z=6-2\rho\cos\theta-2\rho\,\mathrm{sen}\,\theta} |\rho|\,dz}_{g(\theta,\rho)}\right)d\rho\right]}_{h(\theta)}d\theta$$

$g(\theta,\rho) = 4\rho-\rho^3$; $h(\theta)=4$; $I = 8\pi$

4.h) **CART.**



$z=14$

$z=14$

cilindro $z=y^2+2$

$x=3$

cilindro $z=3y^2+2$

$$I = 2\int_{x=0}^{x=3}\underbrace{\left[\int_{z=2}^{z=14}\left(\underbrace{\int_{y=\sqrt{\frac{z-2}{3}}}^{y=\sqrt{z-2}} dy}_{g(x,z)}\right)dz\right]}_{h(x)}dx$$

$g(x,z) = \left(1-\frac{1}{\sqrt{3}}\right)\sqrt{z-2}$; $h(x) = 16(\sqrt{3}-1)$; $I = 96(\sqrt{3}-1)$

4.i) Dibujamos el corte por el plano ρ-z en cilíndricas **CIL**

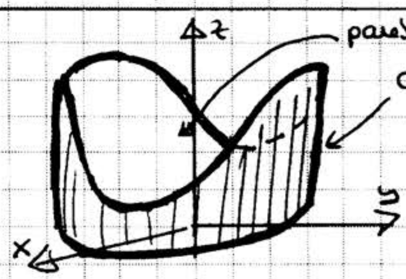
parábola $z=1+\rho^2$

hipérbola $\rho^2-z^2=1$

$$I = \int_{\theta=0}^{\theta=2\pi}\underbrace{\left[\int_{z=0}^{z=1}\left(\underbrace{\int_{\rho=0}^{\rho=\sqrt{1+z^2}} |\rho|\,d\rho}_{g_1(\theta,z)}\right)dz + \int_{z=1}^{z=5}\left(\underbrace{\int_{\rho=\sqrt{z-1}}^{\rho=\sqrt{1+z^2}} |\rho|\,d\rho}_{g_2(\theta,z)}\right)dz\right]}_{h(\theta)}d\theta$$

$g_1(\theta,z) = \frac{1}{2}(1+z^2)$; $g_2(\theta,z) = \frac{1}{2}\left((1+z^2)-(z-1)\right)$; $h(\theta) = \frac{58}{3}$; $I = \frac{116\pi}{3}$

4.j) **CIL**



paraboloide elíptico $z = x^2/4+y^2$

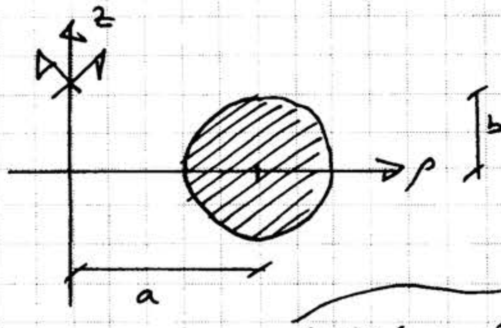
cilindro $x^2+y^2=4$

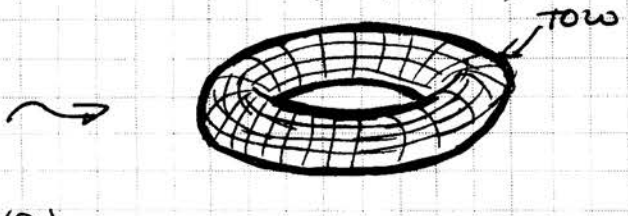
$$I = 4\int_{\theta=0}^{\theta=\pi/2}\underbrace{\left[\int_{\rho=0}^{\rho=2}\left(\underbrace{\int_{z=0}^{z=\rho^2\left(\frac{\cos^2\theta}{4}+\mathrm{sen}^2\theta\right)} |\rho|\,dz}_{g(\theta,\rho)}\right)d\rho\right]}_{h(\theta)}dz$$

$g(\theta,\rho) = \rho^3/4\,(1+3\,\mathrm{sen}^2\theta)$; $h(\theta) = 1+3\,\mathrm{sen}^2\theta$; $I = 5\pi$

4.k) Corte según el plano $\rho - z$ **CIL**

$* \; V=(\pi b^2)(2\pi a)$



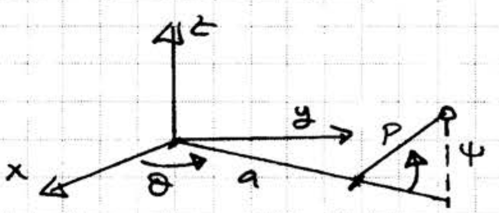


__Cilíndricas__

$$I=\int_{\theta=0}^{\theta=2\pi}\left[\overbrace{\int_{\rho=a-b}^{\rho=a+b}\left(\underbrace{\int_{z=-\sqrt{b^2-(\rho-a)^2}}^{z=+\sqrt{b^2-(\rho-a)^2}}|\rho|\,dz}_{g(\theta,\rho)}\right)d\rho}^{h(\theta)}\right]d\theta$$

$g(\theta,\rho)=2\rho\sqrt{b^2-(\rho-a)^2}$; $h(\theta)=\pi b^2 a$; $I=2\pi^2 b^2 a$

__Curvilíneas Específicas__



$$\bar{u}=\begin{Bmatrix}\theta\\ \psi\\ \rho\end{Bmatrix} \quad \begin{cases} x=(a+\rho\cos\psi)\cos\theta\\ y=(a+\rho\cos\psi)\,\mathrm{sen}\,\theta\\ z=\quad\rho\,\mathrm{sen}\,\psi\end{cases}$$

$$\bar{r}=\begin{Bmatrix}x\\ y\\ z\end{Bmatrix} \quad \underset{\sim}{y}=\frac{d\bar{r}}{d\bar{u}}=\begin{bmatrix}-(a+\rho\cos\psi)\,\mathrm{sen}\,\theta & -\rho\,\mathrm{sen}\,\psi\cos\theta & \cos\psi\cos\theta\\ (a+\rho\cos\psi)\cos\theta & -\rho\,\mathrm{sen}\,\psi\,\mathrm{sen}\,\theta & \cos\psi\,\mathrm{sen}\,\theta\\ 0 & \rho\cos\psi & \mathrm{sen}\,\psi\end{bmatrix}$$

$$\Rightarrow dV=|\det(\underset{\sim}{y})|\,d\theta\,d\psi\,d\rho=|\rho(a+\rho\cos\psi)|\,d\theta\,d\psi\,d\rho$$

$$I=\int_{\theta=0}^{\theta=2\pi}\left[\underbrace{\int_{\psi=0}^{\psi=2\pi}\left(\underbrace{\int_{\rho=0}^{\rho=b}|\rho(a+\rho\cos\psi)|\,d\rho}_{g(\theta,\psi)}\right)d\psi}_{h(\theta)}\right]d\theta$$

$g(\theta,\psi)=a\,b^2/2+b^3/3\,\cos\psi$; $h(\theta)=\pi b^2 a$; $I=2\pi^2 b^2 a$

4.l) Corte según el plano $\rho - z$ **ESF.**

esfera $\rho^2+z^2=a^2$

cono $\rho=\pm z\,\mathrm{tg}\,\alpha$, $\alpha\in(0,\pi/2)$

$$I=\int_{\theta=0}^{\theta=2\pi}\left[\overbrace{\int_{\varphi=\alpha}^{\varphi=\pi-\alpha}\left(\underbrace{\int_{r=0}^{r=a}|r^2\,\mathrm{sen}\,\varphi|\,dr}_{g(\theta,\varphi)}\right)d\varphi}^{h(\theta)}\right]d\theta$$

$g(\theta,\varphi)=a^3/3\,\mathrm{sen}\,\varphi$; $h(\theta)=2/3\,a^3\cos\alpha$; $I=4/3\,\pi a^3\cos\alpha$

4.m) Corte según el plano $\rho - z$ **ESF.**

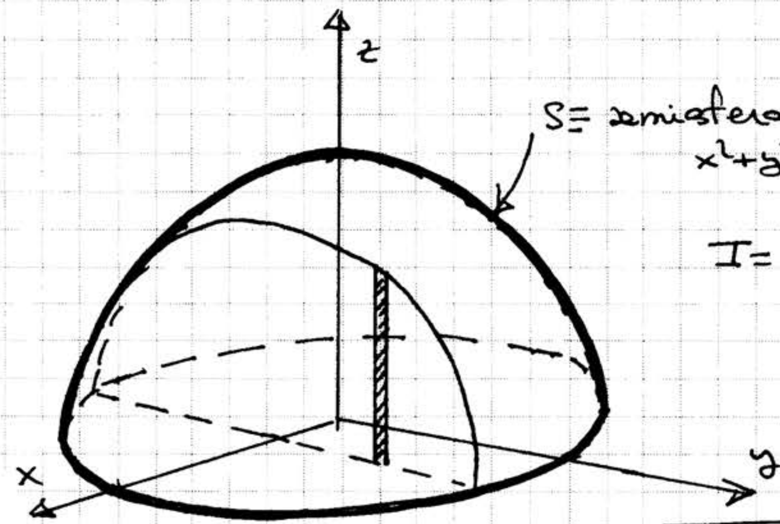
$* \; V=(\pi a^2)(2\pi a)$

$$I=\int_{\theta=0}^{\theta=2\pi}\left[\overbrace{\int_{\varphi=0}^{\varphi=\pi}\left(\underbrace{\int_{r=0}^{r=2a\,\mathrm{sen}\,\varphi}|r^2\,\mathrm{sen}\,\varphi|\,dr}_{g(\theta,\varphi)}\right)d\varphi}^{h(\theta)}\right]d\theta$$

$g(\theta,\varphi)=8/3\,a^3\,\mathrm{sen}^4\varphi$; $h(\theta)=a^3\pi$; $I=2\pi^2 a^3$

5) $$I = \int_{x=-1}^{x=1} \left[\int_{y=-\sqrt{1-x^2}}^{y=+\sqrt{1-x^2}} \left(\int_{z=0}^{z=\sqrt{1-x^2-y^2}} (x^2+y^2)\, dz \right) dy \right] dx$$

Dominio de Integración



S≡ semiesfera de radio 1
$x^2+y^2+z^2=1$

$$I = \iiint_S (x^2+y^2)\, dV \quad (*)$$

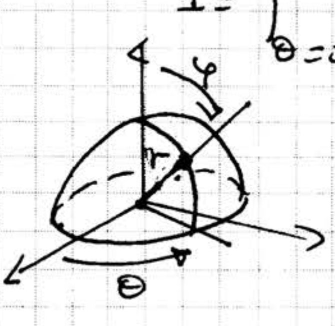
CARTESIANAS

$$I = \int_{x=-1}^{x=1} \underbrace{\left[\int_{y=-\sqrt{1-x^2}}^{y=+\sqrt{1-x^2}} \left(\underbrace{\int_{z=0}^{z=\sqrt{1-x^2-y^2}} (x^2+y^2)\, dz}_{g(x,y)} \right) dy \right]}_{h(x)} dx$$

$g(x,y) = (x^2+y^2)\sqrt{1-(x^2+y^2)}$; $h(x) = \pi/_8\,(1-x^2)(1+3x^2)$; $I = \frac{4}{15}\pi$

ESFERICAS

$$I = \int_{\theta=0}^{\theta=2\pi} \underbrace{\left[\int_{\varphi=0}^{\varphi=\pi/2} \left(\underbrace{\int_{r=0}^{r=1} (r \operatorname{sen}\varphi)^2 \,|r^2 \operatorname{sen}\varphi|\, dr}_{g(\theta,\varphi)} \right) d\varphi \right]}_{h(\theta)} d\theta$$



$g(\theta,\varphi) = 1/_5 \operatorname{sen}^3\varphi$; $h(\theta) = 2/_{15}$; $I = \frac{4}{15}\pi$

(*) NOTA: es el momento de inercia respecto al eje z de la semiesfera de radio 1.
Para la esfera completa: $I_z = 2I_{xx}$, $I_O = 3I_{xx}$, $I_O = \int_0^R r^2(4\pi r^2)\,dr = 4/_5\,\pi R^5 \Rightarrow I_z = \frac{8\pi R^5}{15}$
$1/_2$ esfera $\Rightarrow I_z = \frac{4}{15}\pi R^5$