Infinitesimal Calculus 2. Academic year 2023-24.

Next to the title of each unit, the approximate number of theory (T) and practice (P) lessons dedicated to it is indicated in parentheses.
The course notes can downloaded from the web page. In some sections there are also other supplementary documents. The indication (**o**) means that these documents -usually proofs- are of optional consultation. With (**i**) it is indicated that the documents -usually solved exercises- are considered important for the understanding of the topic.

Unit I. Integration (3-T, 8-P)

1.- Primitive of a function.
   - 1.1- Definition.
   - 1.2- Primitives of discontinuous functions.
   - 1.3- Necessary condition for the existence of primitive.

2.- La Riemann integral.
   - 2.1- Partition of an interval.
   - 2.2- Darboux sums.
   - 2.3- Riemann integrable function.
   - 2.4- Necessary and sufficient condition of integrability.
   - 2.5- Three sufficient conditions of integrability (**o**).
   - 2.6- Properties of the Riemann integral.

3.- Intermediate value theorem.

4.- First fundamental theorem of calculus.
   - 4.1- The integral function.
   - 4.2- Theorem.
   - 4.3- Corollary. Barrow rule.

5.- Second fundamental theorem of Calculus.

6.- Improper integrals.

7.- Applications: areas, volumes, arcs and surfaces of revolution.

Unit II. Vector functions (9-T, 9-P)

1.- Introduction. Types of functions.

2.- Euclidean space.
   - 2.1- Ordinary scalar product.
   - 2.2- Euclidean norm.
   - 2.3- Euclidean distance.

3.- Vector functions of a real variable.
   - 3.1- Limit.
   - 3.2- Continuity.
   - 3.3- Differenctiability.

4.- Real functions of a vector variable.
   - 4.1- Limit (**i**).
   - 4.2- Continuity.
   - 4.3- Directional and partial derivatives.
   - 4.4- Differential.
   - 4.5- Gradient and contour lines. Geometric interpretation.
   - 4.6- Differentiability theorems.

5.- Vector functions of a vector variable.

 5.1- Limit.
 5.2- Continuity.
 5.3- Differentiability.

6.- Composition of functions.

 6.1- Composite function. Continuity and differentiability.
 6.2- Derivative of the composite function. Chain rule (**o**).

7.- Higher-order derivatives.

 7.1- Definition.
 7.2- Cross partial derivatives.
 7.3- Higher-order differentials.

8.- Taylor expansion.

 8.1- General expression.
 8.2- Matrix expression.

9.- Local extrema (**i**).

 9.1- Necessary condition.
 9.2- Sufficient condition.
 9.3- Determination of the type of quadratic form.
 9.4- Search of extrema. Summary and examples.

10.- Implicit function.

 10.1- Definition.
 10.2- Existence and differentiability theorem for two variables.
 10.3- Generalization of the theorem to vector functions (**i**).

11.- Constrained extrema.

 11.1- Constraints in explicit form.
 11.2- Constraints in implicit form (**i**).

Unit III. Series of real numbers (5-T, 5-P)

1.- Definition.
2.- Geometric series.
3.- Necessary condition of convergence.
4.- Properties of the series.
5.- Cauchy's criterion of convergence.
6.- Series of positive terms. Convergence criteria.

 6.1.- Majorant and minorant.
 6.2.- Riemann series.
 6.3.- Comparison of series.
 6.4.- Root test (Cauchy-Hadamard).
 6.5.- Ratio test (D'Alembert).
 6.6.- Raabe's test.
 6.7.- Logarithmic test.
 6.8.- Condensation test.

7.- Series of positive and negative terms.

 7.1.- Absolute and unconditional convergence and divergence.
 7.2.- Riemann's theorem.
 7.3.- Dirichlet's theorem.
 7.4.- Alternating series. Leibnitz's theorem.

8.- Methods for the summation of series (**i**).

   8.1.- By decomposition.
   8.2.- Using the harmonic series.
   8.3.- Using the series expansion of e^x.
   8.4.- Hypergeometric series.
   8.5.- Convergence and sum of a series. Summary.

Unit IV. Sequences and series of functions (3-T, 4-P)

1.- Sequences of functions.

   1.1.- Distance between functions.
   1.2.- Sequence of functions.
   1.3.- Pointwise convergence.
   1.4.- Uniform convergence.
   1.5.- Sequence of continuous functions (**o**).

2.- Series of functions.

   2.1.- Definition.
   2.2.- Pointwise and uniform convergence.
   2.3.- Criterion of the majorant.
   2.4.- Series of continuous functions.
   2.5.- Integration of a series of functions.
   2.6.- Derivation of a series of functions.

3.- Power series.

   3.1.- Definition.
   3.2.- Cauchy-Hadamard's theorem.
   3.3.- Continuity, derivation and integration (**o**).
   3.4.- Abel's theorems.
   3.5.- Power series expansion of a function. Taylor series.

Unit V. Complex numbers (2-T, 2-P)

1.- Introduction.
2.- Definition. Binomic form. Basic operations.
3.- Trigonometric form. Graphic representation.
4.- Conjugate complex, additive and multiplicative inverse. Quotient.
5.- Exponential of a complex number. Euler's formula.
6.- Power of natural exponent. Moivre's formula.
7.- $n-th$ root of a complex.
8.- Fundamental theorem of Algebra.

A Coruña, January 29, 2.024