## Infinitesimal Calculus 2

## Integrals 3

## Arc length and surface area

1.- Find the length of the indicated arcs:
a) $y=x^{3 / 2}$, between $x=0$ and $x=5$.
b) $x=3 y^{3 / 2}-1$, between $y=0$ and $y=4$.
c) $24 x y=x^{4}+48$, between $x=2$ and $x=4$.
d) $y=\frac{1}{2} a\left(e^{\frac{x}{a}}+e^{-\frac{x}{a}}\right)$, between $x=0$ and $x=a$.
e) $y=\ln \cos x$, between $x=0$ and $x=a$, being $a<\frac{\pi}{2}$.
f) $y=\ln x$, between $x=1$ and $x=2 \sqrt{2}$.
g) $x=\frac{1}{2} y^{2}-\frac{1}{4} \ln y$, between $y=1$ and $y=e$.
h) $y=a \ln \frac{a^{2}}{a^{2}-x^{2}}$, between $x=0$ and $x=b$, being $b<a$.
2.- Find the length of the following arcs:
a) $x=t^{2} ; y=t^{3}$; between $t=0$ and $t=4$.
b) $x=t-\sin t ; y=1-\cos t$; between $t=0$ and $t=2 \pi$.
c) $x=e^{t} \cos t ; y=e^{t} \sin t$; between $t=0$ and $t=4$.
d) $x=a(\cos t+t \sin t) ; y=a(\sin t-t \cos t)$; between $t=0$ and $t=2 \pi$.
3.- Obtain the area of the surface generated by rotating the indicated arc about the given axis:
a) $y^{2}=12 x$, between $x=0$ and $x=3$, about the $x$-axis.
b) $x=y^{3}$, between $y=0$ and $y=1$, about the $y$-axis.
c) $y=a \cosh \frac{x}{a}$, between $x=-a$ and $x=a$, about the $x$-axis.
d) $x+y^{2}-3=0$, for $x \geq 0$, about the $x$-axis.
4.- Find the area of the surface generated by rotating the indicated arc about the given axis:
a) $x=a \cos ^{3} t ; y=a \sin ^{3} t$; about the $x$-axis.
b) $x=a(t-\sin t) ; y=a(1-\cos t)$; between $t=0$ and $t=2 \pi$ about the $x$-axis.
5.- Find the length of the circumference and the surface area of the sphere:
a) Using explicit cartesian coordinates.
b) Using parametric cartesian coordinates.

