1.– Determine the volumes obtained by rotating the region bounded by the following curves, about the indicated axes:

a) $y^2 = 8x$; $y = 0$; $x = 2$.

1. About the x-axis.
2. About the y-axis.
3. About the line $x = 2$.

b) $y = 2x - x^2$; $y = 0$.

1. About the x-axis.
2. About the y-axis.
3. About the line $y = 2$.

c) $x^2 + y^2 = 4$, About the line $x = 3$.

d) $y = -x^2 - 2x + 3$; $x + y - 1 = 0$.

1. About the line $x = 1$.
2. About the x-axis.

e) $y = \dfrac{bx^2}{a^2}$; $y = \dfrac{b\,|x|}{a}$; $a > b > 0$.

1. About the x-axis.
2. About the y-axis.

f) $y = \sin x$; $y = 0$; $x = 0$; $x = \pi$.

1. About the x-axis.
2. About the y-axis.

g) $x = t - \sin t$, $y = 1 - \cos t$, $t \in [0, 2\pi]$; $y = 0$.

1. About the x-axis.
2. About the y-axis.

h) $x = \sin^3 t$, $y = \cos^3 t$, $t \in [0, 2\pi]$.

1. About the x-axis.
2. About the y-axis.

2.– Calculate the volume of the solids bounded by the following surfaces:

a) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1.$

b) $z = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$; $z = c$.

c) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} + 1 = 0$; $z = 2c$; $z = -2c$.

d) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 1$; $z = c$; $z = -c$.

e) $\dfrac{x^2}{a^2} + \dfrac{y^2}{z^2} = 1$; $z = 0$; $z = a$.

f) $x + y + z^2 = 1$ (in the 1st octant).

3.– Solve the following problems by integration:

a) Consider a cylinder of axis OZ and circular cross-section of radius $4\,\mathrm{m}$. Obtain the volume enclosed outside the cylinder and inside the surface of equation

$$\frac{x^2}{2} + \frac{y^2}{2} = z^2$$

b) In a sphere of radius $5\,\mathrm{m}$. a hole is made in the shape of a circular cylinder of radius $4\,\mathrm{m}$. The axis of the cylindrical drill is a diameter of the sphere. Calculate the volume of the resulting solid.

c) Calculate the volume enclosed between the surfaces S_1 and S_2.

$$S_1 : \; x^2 + y^2 - z - 1 = 0 \,; \quad S_2 : \; x^2 + y^2 + z - 3 = 0$$

d) Consider the paraboloid of equation $x^2 + y^2 + z = 9$ and the cube with the base on the plane XY and the center at the point $C(0,0,3)$. Calculate the volume common to both solids.

e) Consider a sphere of radius 2 meters whose center is 4 meters above the horizontal plane. A tank is formed by the part of the sphere situated above the plane $z = 3$. The tank is filled with a liquid whose density $\rho(z)$ is a function of height. The thickness of the walls is assumed to be negligible. It is requested to obtain:

1. The integral expression to calculate the mass of the liquid contained in the tank.
2. The volume of the liquid in m^3.

f) We want to know the volume and cost of an excavation. The excavated volume is shaped like a truncated cone. Its major base is a circle located on the XY plane, which corresponds to the ground plane. The excavation is $3\,m$ deep and its bottom is horizontal. The equation of the surface of the cone is

$$x^2 + y^2 - (z+5)^2 = 0$$

The cost per m^3 of excavation is a function of depth and is given by $f(z)$. It is requested:

1. To obtain the integral expression to calculate the cost of the excavation.
2. Calculate the excavation volume in m^3.