

## Unit IV. Lessons distribution and self-assessment questions.

- Lesson 1. Sections 1 to 2.1.
  1. Is the distance between two functions the maximum of the point-to-point distances?
  2. Can we form a functional sequence with discontinuous functions? Give an example.
  3. If the functions  $f_n$  of a sequence are continuous, will its limit function be continuous?
  4. Give an example of a sequence of continuous  $f_n$  that converge to a discontinuous  $\phi$ .
  5. Give an example of a sequence of discontinuous  $f_n$  that converge to a continuous  $\phi$ .
  
- Lesson 2. Sections 2.2 to 3.3.
  1. What happens to a functional series defined in  $I$ , if we replace  $x$  by one of its values on  $I$ ?
  2. Is there always the sum function of a geometric series of ratio  $x$ ?
  3. Under what conditions is the integral of the sum the sum of the series of integrals?
  4. Does every sequence of real numbers have a limit, finite or infinite?
  5. Can the interval of convergence of a power series take any form?
  
- Lesson 3. Sections 3.4 and 3.5.
  1. If the radius of convergence of  $\sum a_n x^n$  is  $r > 0$ , what do we know about its sum  $S(x)$ ?
  2. If  $\sum a_n x^n$  converges at  $x = r > 0$ , what type of convergence will there be on  $\left[\frac{r}{3}, \frac{r}{2}\right]$ ?
  3. What do we use Abel's second theorem for?
  4. If  $\sum a_n$  is a power series expansion of  $f$ , does its interval of convergence coincide with that of existence of  $f$ ?
  5. Does any function  $f \in C^\infty$  have a Taylor series expansion?