Unit II. Lessons distribution and self-assesment questions

- Lesson 1. Sections 1 to 3.

  1. State the three properties of the ordinary scalar product.
  2. What is an euclidean space?
  3. What is the relation between the euclidean norm and the modulus of a vector?
  4. A vector function has a limit if and only if all its components have a limit. True?
  5. What is the continuity condition for vector functions?

- Lesson 2. Sections 4.1 and 4.2.

  1. Does the functional limit of $f(x, y)$ depend on the direction of approaching the point?
  2. Can the limit of $f(x, y)$ exist if the directionals do not exist?
  3. If all the directional limits exist and coincide, what can we say of the functional limit?
  4. Can we calculate the limit of $f(x, y)$ without first calculating the directionals?
  5. The directional derivative of $f(\vec{x})$, is the limit of $\Delta f/\Delta\vec{x}$, when $\lambda \to 0$ (calculating Δf between $\vec{x} = \vec{a}$ and $\vec{x} = \vec{a} + \lambda\vec{\omega}$). True?

- Lesson 3. Sections 4.3 to 4.5.

  1. Explain the relation between directional and partial derivatives.
  2. How do we obtain the partial derivatives of a polynomial function of 3 variables?
  3. How can we find the directional derivatives of a differentiable function?
  4. Which components has the total derivative of a differentiable function of n variables?
  5. Which vector indicates the direction of null variation of a differentiable function?

- Lesson 4. Sections 4.6 and 5.

  1. State a sufficient, not necessary, condition of differentiability.
  2. State a necessary, not sufficient, condition of differentiability.
  3. If $f(x, y, z)$ has partial derivatives, then it is differentiable. True or false?
  4. If $f(x, y, z)$ is not continuous, then it has no partial derivatives. True or false?
  5. ¿What is the Jacobian matrix of a function $\vec{f}(\vec{x})$?

- Lesson 5. Sections 6 to 7.2.

  1. ¿Is the composition of differentiable functions continuous?
  2. Can the composition of continuous functions be differentiable?
  3. If $\vec{g} \circ \vec{f}$ is not differentiable, at least one of the two functions is not. True?
  4. How many second derivatives does $f(x, y)$ have? And how many third derivatives?
  5. Does the third order derivatives of $f(x, y)$ depend on the order of derivation?

- Lesson 6. Sections 7.3 and 8.

  1. What is a square in simbolic form?
  2. When obtaining the $d^4 f$, find the coefficients of the fourth derivatives of $f(x,y)$, if the cross derivatives coincide.
  3. Is the Hessian matrix of f made up of the partial derivatives of the function?
  4. The Hessian matrix is symmetric. True or false?
  5. How is the Taylor polynomial P_k different from the limited Taylor expansion?

- Lesson 7. Section 9.

  1. Are all possible extrema of $f(\vec{x})$ at the critical points?
  2. If $f(x,y)$ has an extremum at $\vec{a}$, what is the value of its partial derivatives at $\vec{a}$?
  3. If the necessary extremum condition is satisfied at $\vec{a}$, what does the $d^2 f(\vec{a})$ indicate?
  4. There is an extremum at $\vec{a}$ if and only if $d^2 f(\vec{a})$ is a definite quadratic form. True?
  5. What is the relation between the determinant of H and that of its congruent diagonal matrix?

- Lesson 8. Section 10.

  1. Consider $F(x,y) = 0$, from which we cannot solve for y as a function of x. When can we ensure that for all x there is a value of y which verifies the equation?
  2. Given an equation that relates x and y, can it be possible to calculate its derivative, even if there is no explicit expression $y = f(x)$?
  3. Give a practical example of applying the implicit function theorem.
  4. If $F(x,y) = x^2 + y^2 - 4 = 0$, which points of the curve defined by the equation do not satisfy the conditions of the implicit function theorem?
  5. Let an equation with three unknowns be $F(x,y,z) = 0$. Under what conditions can we obtain the derivatives of z with respect to x and y without first obtaining the expression $z(x,y)$?

- Lesson 9. Section 11.

  1. If a function of several variables has an extremum at P, can the extremum disappear if we impose a certain condition on some of the variables?
  2. How many unknowns and how many conditions do we consider in the calculation of constrained extremes? To what number are the unknowns reduced to if the conditions are explicit?
  3. What do we call the Lagrange multipliers? How many multipliers are there, if there exist m constraints?
  4. We look for the maxima and minima of the function $u = f(x,y,z)$, subject to the constraints $g_1(x,y,z) = 0$, $g_2(x,y,z)$ =0. Write the expression of the Lagrangian function.
  5. To find out wether the sufficient condition of extremum is satisfied at a critical point, is it always necessary to study the conditions $dg_i = 0$, $i = 1, \ldots m$?