Infinitesimal Calculus 1. Academic year 2023-24.

Next to the title of each unit, the approximate number of theory (T) and practice (P) lessons dedicated to the unit is indicated in parentheses.

The course notes can be downloaded from the web page. In some sections there are also other supplementary documents. The indication (\mathbf{o}) means that these documents -usually proofs- are of optional consultation. With (\mathbf{i}) it is indicated that the documents -usually solved exercises- are considered important for the understanding of the topic.

Unit I. The real number (5-T, 4-P)

1.- Introduction.

- 1.1.- Necessary and sufficient condition.
- 1.2.- Demonstration by *reductio ad absurdum*.
- 2.- Successive extensions of the concept of number.
 - 2.1.- Natural numbers.
 - 2.2.- Integer numbers.
 - 2.3.- Rational numbers.
 - 2.4.- Countable sets (o). Principle of induction (i).
- 3.- Field structure. Ordered field.
 - 3.1.- Definition of field.
 - 3.2.- Order.
 - 3.3.- Ordered field.
 - 3.4.- Bounds and intervals.
 - 3.5.- Absolute value.
- 4.- Sequences in \mathbb{Q} .
 - 4.1.- Definition.
 - 4.2.- Convergent sequence.
 - 4.3.- Cauchy sequences.
- 5.- Properties of \mathbb{Q} .
- 6.- Extension of \mathbb{Q} . The real numbers.
- 7.- Properties of \mathbb{R} (**o**).
- 8.- Operations in \mathbb{R} .

Unit II. Metric spaces (4-T, 4-P)

- 1.- Distance.
 - 1.1.- Definition.
 - 1.2.- Most common metrics.
- 2.- Balls and neighborhoods.
 - 2.1.- Open ball.
 - 2.2.- Closed ball.
 - 2.3.- Neighborhood.
- 3.- Notable points in a metric space.
 - 3.1.- Closure point.
 - 3.2.- Accumulation point.
 - 3.3.- Isolated point.
 - 3.4.- Interior point.
 - 3.5.- Exterior point.
 - 3.6.- Boundary point.

- 4.- Notable sets in a metric space.
 - 4.1.- Closure of A.
 - 4.2.- Set of isolated points.
 - 4.3.- Derived set of A.
 - 4.4.- Interior of A.
 - 4.5.- Exterior of A.
 - 4.6.- Boundary of A.
- 5.- Closed, open and compact sets.
 - 5.1.- Closed set.
 - 5.2.- Open set.
 - 5.3.- Relation between open and closed sets.
 - 5.4.- Compact set.
- 6.- The metric space $(\mathbb{R}, ||)$.
 - 6.1.- Distance.
 - 6.2.- Open and closed sets.
 - 6.3.- Bolzano-Weierstrass theorem.

Unit III. Sequences on \mathbb{R} (5-T, 4-P)

- 1.- Definition and types of sequences.
 - 1.1.- Sequences in \mathbb{R} .
 - 1.2.- Concept of limit.
 - 1.3.- Types of sequences.
- 2.- Properties of the limits.
- 3.- Monotone sequences.
 - 3.1.- Definitions.
 - 3.2.- Theorem of monotone sequences.
 - 3.3.- Sequences of nested intervals.
- 4.- Operations with limits.
 - 4.1.- Addition and subtraction.
 - 4.2.- Product.
 - 4.3.- Inverse.
 - 4.4.- Division.
 - 4.5.- Logarithm.
 - 4.6.- Exponential.
 - 4.7.- Potential-exponential.
 - 4.8.- Indeterminate forms.
- 5.- Convergence criteria.
 - 5.1.- Stolz theorem.
 - 5.2.- Arithmetic mean criterion.
 - 5.3.- Geometric mean criterion.
 - 5.4.- Root law.
- 6.- Infinites and infinitesimals.
 - 6.1.- Definitions.
 - 6.2.- Comparison.
 - 6.3.- Relation between types of infinite.
- 7.- Equivalent sequences.
 - 7.1.- Definition.
 - 7.2.- Properties.
 - 7.3.- Equivalence to the principal parts.

- 8.- Substitution by equivalent sequences.
 - 8.1.- Product and quotient.
 - 8.2.- Logarithm.
 - 8.3.- Potential-exponential.
 - 8.4.- Addition and subtraction.
- 9.- Some practical methods.
 - 9.1.- Using number e.
 - 9.2.- Polynomial expressions.
 - 9.3.- Recurrent sequences.
 - 9.4.- Stirling and trigonometric equivalences.
 - 9.5.- Change of the type of indetermination. Annex. Equivalences table.

Unit IV. Real functions (9-T, 9-P)

- A. General notions.
 - 1.- Concept of function.
 - 2.- Operations with functions.
 - 3.- Types of functions.
- B. Limits of functions.
 - 1.- Limit of a function.
 - 2.- One-sided limits.
 - 3.- Extension of the concept of limit.
 - 4.- Sequential limit. Relation with the limit of a function.
 - 5.- Properties of the limits.
 - 6.- Operations with limits.
 - 7.- Infinites and infinitesimals.
 - 7.1- Definitions.
 - 7.2- Comparison.
 - 7.3- Relation between types of infinite.
 - 8.- Equivalent functions at a point.
 - 9.- Substitution by equivalent functions.
 - 9.1- Product and division.
 - 9.2- Logarithm.
 - 9.3- Potential-exponential.
 - 9.4- Addition and subtraction. Annex. Equivalences table.
- C. Continuity of functions.
 - 1.- Continuous function.
 - 2.- One-sided continuity.
 - 3.- Discontinuities.
 - 4.- Operations with continuous functions.
 - 5.- Continuity of the elementary functions.
 - 6.- Composition of continuous functions.
 - 7.- Theorems of continuous functions.
 - 7.1- Bolzano's theorem.
 - 7.2- Darboux property (intermediate value property).
 - 7.3- Weierstrass theorem.
 - 7.4- Image of a closed interval.
 - 7.5- Image of an interval.

- 8.- Uniform continuity.
 - 8.1- Definition.
 - 8.2- Heine's theorem.
 - 8.3- Composition of uniformly continuous functions.
- D. Differentiability of functions.
 - 1.- Derivability and differentiability.
 - 1.1- Derivable function.
 - 1.2- Differentiable function.
 - 1.3- Relation between both concepts.
 - 1.4- Graphic interpretation.
 - 1.5- Relations between continuity and differentiability.
 - 1.6- Operations with differentiable functions.
 - 2.- Chain rule. Applications.
 - 2.1- Derivative of the composite function.
 - 2.2- Applications of the chain rule.
 - 3.- Derivative of the inverse function.
 - 3.1- Existence of the inverse function.
 - 3.2- Derivative of the inverse function.
 - 4.- Mean value theorems.
 - 4.1- Rolle's theorem.
 - 4.2- Cauchy's theorem.
 - 4.3- Lagrange's theorem (mean value theorem).
 - 4.4- Differentiable monotone functions.
 - 4.5- Constant functions.
 - 5.- The derivative as a limit of derivatives.
 - 6.- L'hôpital rules (i).
 - 7.- Higher order derivatives.
 - 8.- Limited Taylor and McLaurin expansions.
 - 8.1- Limited Taylor expansion of order n.
 - 8.2- Limited MacLaurin expansion of order n (i).
 - 8.3- Lagrange remainder.
 - 8.4- Relative extremes property.
 - 8.5- Applications of the Taylor expansion.
 - 8.6- Expansions deduced from others.
 - 9.- Representation of curves (i).

Unit V. Techniques of integration (2-T, 7-P)

- 1.- Introduction and basic concepts.
- 2.- Hyperbolic functions.
- 3.- Primitive function. Immediate integrals.
- 4.- Semiimmediate integrals and changes of variable (o).
- 5.- Integration by parts.
- 6.- Reduction formulas.
- 7.- Rational functions.
- 8.- Trigonometric integrals. Changes of variable.
- 9.- Irrational integrals. Changes of variable.
- 10.- Table of inmediate integrals.