Cálculo Infinitesimal 1 — Ejercicio 5

Integrales y Curvas (1 h. 30 m.) **diciembre de 2016**

1.- Obtener la fórmula de reducción para $I(n) = \int \arg \mathrm{sh}^n x \, dx$.

- $u = \arg sh^n x \Rightarrow du = n \arg sh^{n-1} x \frac{dx}{\sqrt{x^2+1}}$, $dv = dx \Rightarrow v = x$ $\Big\}$ $I(n) = x \cdot \arg sh^n x - n \underbrace{\int \arg sh^{n-1} x \frac{x}{\sqrt{x^2+1}} dx}_{I_\alpha}$

- $I_\alpha = \int \arg sh^{n-1} x \frac{x}{\sqrt{x^2+1}} dx$

  $u = \arg sh^{n-1} x \Rightarrow du = (n-1) \arg sh^{n-2} x \frac{dx}{\sqrt{x^2+1}}$, $dv = \frac{x}{\sqrt{x^2+1}} dx \Rightarrow v = \sqrt{x^2+1}$ $\Big\}$ $I_\alpha = \sqrt{x^2+1} \arg sh^{n-1} x - (n-1) \underbrace{\int \arg sh^{n-2} x \, dx}_{I(n-2)}$

- $\boxed{I(n) = x \arg sh^n x - n \sqrt{x^2+1} \arg sh^{n-1} x + n(n-1) I(n-2)}$

  $I(0) = \int dx = x$
  $I(1) = \int \arg sh x \, dx = \cdots = -\sqrt{x^2+1}$ $\Big\}$ vale la fórmula gral.

2.- Calcular la primitiva de $\int \frac{\mathrm{sen}\, 2x}{(1+\mathrm{tg}^2 x)^p} dx$, $p \in \mathbb{R}$.

- $\boxed{I} = \int \frac{2 \operatorname{sen} x \cdot \cos x}{(1/\cos^2 x)^p} = 2 \int \underbrace{(\cos x)}_{t}{}^{2p+1} \cdot \underbrace{\operatorname{sen} x \cdot dx}_{-dt} = -2 \frac{\cos^{2p+2} x}{2p+2} = \boxed{\frac{-\cos^{2p+2} x}{p+1} + C \quad (p \neq -1)}$

- Si $p = -1$: $I = 2 \int \frac{\operatorname{Sen} x}{\cos x} dx = -2 L|\cos x| + C$

3.- Calcular la primitiva de $\int \frac{2+\sqrt{x}}{1+\sqrt{x}} dx$.

$x = t^2 \Rightarrow dx = 2t\,dt$

$I = 2 \int \frac{2+t}{1+t} t \, dt = 2 \int \frac{2t+t^2}{1+t} dt$; $\quad \begin{array}{r|l} t^2 + 2t & t+1 \\ -t^2 - t & t+1 \\ \hline t & \\ -t - 1 & \\ \hline -1 & \end{array}$ $\Big\}$ $I = 2 \int \left(t + 1 - \frac{1}{t+1}\right) dt \Rightarrow$

$\boxed{I} = t^2 + 2t - 2 L|t+1| = \boxed{x + 2\sqrt{x} - 2 L|1+\sqrt{x}| + C}$

4.- Estudiar la curva de ecuación $y = \frac{x^2-4}{(x+1)(x-1)}$ y representarla aproximadamente.

- $y = f(x)$ existe $\forall x \neq \pm 1$ y tiene ceros en $x = \pm 2$
- $f(-x) = f(x) \Rightarrow$ simétrica respecto OY.
- Asint. Vert: Si $x \to \pm 1 \Rightarrow f(x) \to \pm\infty$
- Asint. Horiz: Si $x \to \pm\infty \Rightarrow f(x) \to 1^-$
  $(x^2 - 4 < x^2 - 1)$
- Max y Min: $f'(x) = 0$

  $f(x) = \frac{x^2-4}{x^2-1} = \frac{x^2-1-3}{x^2-1} = 1 - \frac{3}{x^2-1}$

  $f'(x) = \frac{6x}{(x^2-1)^2} = 0 \Rightarrow x = 0 \Rightarrow f(0) = 4$

  $(x = \pm\varepsilon \Rightarrow f(x) > 0 \Rightarrow$ Mínimo$)$

  $f''(x) = \frac{6(x^2-1)^2 - 6x(x^2-1)2x}{(x^2-1)^4} \Rightarrow f''(0) = 6 > 0$
  Mínimo