1.– Solve the following integrals:

a) $\displaystyle\int \sin 3x \sin 2x \; dx$ **Sol:** $\dfrac{1}{2}\sin x - \dfrac{1}{10}\sin 5x + C$

b) $\displaystyle\int \cos 4x \cos 2x \; dx$ **Sol:** $\dfrac{1}{4}\sin 2x + \dfrac{1}{12}\sin 6x + C$

c) $\displaystyle\int \sin 3x \cos 5x \; dx$ **Sol:** $\dfrac{1}{4}\cos 2x - \dfrac{1}{16}\cos 8x + C$

d) $\displaystyle\int \cos 2x \sin 3x \; dx$ **Sol:** $-\dfrac{1}{2}\cos x - \dfrac{1}{10}\cos 5x + C$

2.– Find the following integrales:

a) $\displaystyle\int \frac{\cos^4 x}{\sin^5 x} \; dx$ **Sol:** $\dfrac{3}{16}\ln\left|\dfrac{1-\cos x}{1+\cos x}\right| + \dfrac{3\cos x - 5\cos^3 x}{8\sin^4 x} + C$

b) $\displaystyle\int \sin^2 x \cos^5 x \; dx$ **Sol:** $\dfrac{1}{3}\sin^3 x - \dfrac{2}{5}\sin^5 x + \dfrac{1}{7}\sin^7 x + C$

c) $\displaystyle\int \frac{1}{\sin^2 x \cos^2 x} \; dx$ **Sol:** $\tan x - \operatorname{cotan} x + C$

d) $\displaystyle\int \frac{1}{\sin^3 x \cos x} \; dx$ **Sol:** $\ln|\tan x| - \dfrac{1}{2}\operatorname{cotan}^2 x + C$

3.– Obtain the primitives:

a) $\displaystyle\int \frac{1}{\cos x - \sin x} \; dx$ **Sol:** $\dfrac{1}{\sqrt{2}}\ln\left|\dfrac{\sqrt{2}+1+\tan\frac{x}{2}}{\sqrt{2}-1-\tan\frac{x}{2}}\right| + C$

b) $\displaystyle\int \frac{5\cos x + 6}{2\cos x + \sin x + 3} \; dx$ **Sol:** $\ln|2\cos x + \sin x + 3| + 2x + C$

c) $\displaystyle\int \frac{1+\tan x}{1-\tan x} \; dx$ **Sol:** $-\ln|\cos x - \sin x| + C$

d) $\displaystyle\int \frac{3\sin x + 2\cos x}{2\sin x + 3\cos x} \; dx$ **Sol:** $-\dfrac{5}{13}\ln|2\sin x + 3\cos x| + \dfrac{12}{13}x + C$

4.– Solve the following integrals:

a) $\displaystyle\int \frac{\sin x}{1+\cos x + \cos 2x} \; dx$ **Sol:** $\ln|2 + \sec x| + C$

b) $\displaystyle\int \frac{\tan x}{1+\sin^2 x \tan^2 x} \; dx$ **Sol:** $\dfrac{1}{\sqrt{3}}\arctan\left(\dfrac{2\tan^2 + 1}{\sqrt{3}}\right) + C$

c) $\displaystyle\int \frac{\sin 2x}{(2+\sin x)^2} \; dx$ **Sol:** $\ln(2+\sin x)^2 + \dfrac{4}{2+\sin x} + C$

d) $\displaystyle\int \frac{1}{\sin x + \sin 2x} \; dx$ **Sol:** $\dfrac{1}{6}\ln|1-\cos x| + \dfrac{1}{2}\ln|1+\cos x| - \dfrac{2}{3}\ln|1+2\cos x| + C$

e) $\displaystyle\int \frac{\cos x}{\cos 2x} \; dx$ **Sol:** $\dfrac{1}{2\sqrt{2}}\ln\left|\dfrac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| + C$

f) $\displaystyle\int \frac{1}{\cos x \cos 2x} \; dx$ **Sol:** $\dfrac{1}{2}\ln\left|\dfrac{1-\sin x}{1+\sin x}\right| + \dfrac{1}{\sqrt{2}}\ln\left|\dfrac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}\right| + C$

g) $\displaystyle\int \frac{\cos^2 x}{4\cos^2 x + \sin^2 x} \; dx$ **Sol:** $\dfrac{1}{3}\left[x - \dfrac{1}{2}\arctan\left(\dfrac{\tan x}{2}\right)\right] + C$

h) $\displaystyle\int \frac{\sin 2x}{1+\sin^2 x} \; dx$ **Sol:** $\ln\left(1+\sin^2 x\right) + C$

5.– Solve, depending on the values of parameter $a \in \mathbb{R}$, the following integrals:

a) $\displaystyle\int \frac{1}{1+a\cos^2 x}\,dx$

Sol: $a=-1,\ I=-\operatorname{cotan} x + C$

$$a>-1,\ I=\frac{1}{\sqrt{1+a}}\arctan\frac{\tan x}{\sqrt{1+a}}+C$$

$$a<-1,\ I=\frac{-1}{\sqrt{-1-a}}\operatorname{argth}\frac{\tan x}{\sqrt{-1-a}}+C$$

$$I=\frac{-1}{2\sqrt{-1-a}}\ln\left|\frac{\sqrt{-1-a}+\tan x}{\sqrt{-1-a}-\tan x}\right|+C$$

b) $\displaystyle\int \frac{1}{1+a\sin^2 x}\,dx$

Sol: $a=-1,\ I=\tan x + C$

$$a>-1,\ I=\frac{1}{\sqrt{1+a}}\arctan\left(\sqrt{1+a}\tan x\right)+C$$

$$a<-1,\ I=\frac{1}{\sqrt{-1-a}}\operatorname{argth}\left(\sqrt{-1-a}\tan x\right)+C$$

$$I=\frac{1}{2\sqrt{-1-a}}\ln\left|\frac{1+\sqrt{-1-a}\tan x}{1-\sqrt{-1-a}\tan x}\right|+C$$

c) $\displaystyle\int \frac{1}{2-a\sin^2 x}\,dx$

Sol: $a=2,\ I=\dfrac{1}{2}\tan x + C$

$$a<2,\ I=\frac{1}{\sqrt{2}\sqrt{2-a}}\arctan\left(\frac{\sqrt{2-a}\tan x}{\sqrt{2}}\right)+C$$

$$a>2,\ I=\frac{1}{\sqrt{2}\sqrt{a-2}}\operatorname{argth}\left(\frac{\sqrt{a-2}\tan x}{\sqrt{2}}\right)+C$$

$$I=\frac{1}{\sqrt{8}\sqrt{a-2}}\ln\left|\frac{\sqrt{2}+\sqrt{a-2}\tan x}{\sqrt{2}-\sqrt{a-2}\tan x}\right|+C$$

d) $\displaystyle\int \frac{1}{3-a\cos^2 x}\,dx$

Sol: $a=3,\ I=\dfrac{1}{3}\operatorname{cotan} x + C$

$$a<3,\ I=\frac{1}{\sqrt{3}\sqrt{3-a}}\arctan\left(\frac{\sqrt{3}\tan x}{\sqrt{3-a}}\right)+C$$

$$a>3,\ I=\frac{-1}{\sqrt{3}\sqrt{a-3}}\operatorname{argth}\left(\frac{\sqrt{3}\tan x}{\sqrt{a-3}}\right)+C$$

$$I=\frac{-1}{\sqrt{12}\sqrt{a-3}}\ln\left|\frac{\sqrt{a-3}+\sqrt{3}\tan x}{\sqrt{a-3}-\sqrt{3}\tan x}\right|+C$$

6.– Obtain the following primitives:

a) $\displaystyle\int \sin^5 x\,\sqrt[3]{\cos x}\,dx$

Sol: $\displaystyle -3\sqrt[3]{\cos x}\left(\frac{\cos^5 x}{16}-\frac{\cos^3 x}{5}+\frac{\cos x}{4}\right)+C$

b) $\displaystyle\int \frac{1}{\sqrt{\sin^3 x\cos^5 x}}\,dx$

Sol: $\displaystyle \frac{2}{3}\sqrt{\tan^3 x}-\frac{2}{\sqrt{\tan x}}+C$

c) $\displaystyle\int \sin^3\left(\frac{x}{2}\right)\cos^5\left(\frac{x}{2}\right)\,dx$

Sol: $\displaystyle \frac{1}{4}\cos^8\frac{x}{2}-\frac{1}{3}\cos^6\frac{x}{2}+C$

d) $\displaystyle\int \frac{1}{\sin\left(\frac{x}{2}\right)\cos^3\left(\frac{x}{2}\right)}\,dx$

Sol: $\displaystyle 2\ln\left|\tan\frac{x}{2}\right|+\tan^2\frac{x}{2}+C$

e) $\displaystyle\int \operatorname{cotan}^3 x\operatorname{cosec}^4 x\,dx$

Sol: $\displaystyle \frac{1}{4}\operatorname{cosec}^4 x-\frac{1}{6}\operatorname{cosec}^6 x+C$

f) $\displaystyle\int \operatorname{cotan}^3 x\operatorname{cosec}^3 x\,dx$

Sol: $\displaystyle \frac{1}{3}\operatorname{cosec}^3 x-\frac{1}{5}\operatorname{cosec}^5 x+C$
