1.– Resolver las siguientes integrales:

a) $\displaystyle\int \operatorname{sen} 3x \operatorname{sen} 2x \; dx$ **Sol:** $\displaystyle\frac{1}{2}\operatorname{sen} x - \frac{1}{10}\operatorname{sen} 5x + C$

b) $\displaystyle\int \cos 4x \cos 2x \; dx$ **Sol:** $\displaystyle\frac{1}{4}\operatorname{sen} 2x + \frac{1}{12}\operatorname{sen} 6x + C$

c) $\displaystyle\int \operatorname{sen} 3x \cos 5x \; dx$ **Sol:** $\displaystyle\frac{1}{4}\cos 2x - \frac{1}{16}\cos 8x + C$

d) $\displaystyle\int \cos 2x \operatorname{sen} 3x \; dx$ **Sol:** $\displaystyle-\frac{1}{2}\cos x - \frac{1}{10}\cos 5x + C$

2.– Hallar las siguientes integrales:

a) $\displaystyle\int \frac{\cos^4 x}{\operatorname{sen}^5 x} \; dx$ **Sol:** $\displaystyle\frac{3}{16}\ln\left|\frac{1-\cos x}{1+\cos x}\right| + \frac{3\cos x - 5\cos^3 x}{8\operatorname{sen}^4 x} + C$

b) $\displaystyle\int \operatorname{sen}^2 x \cos^5 x \; dx$ **Sol:** $\displaystyle\frac{1}{3}\operatorname{sen}^3 x - \frac{2}{5}\operatorname{sen}^5 x + \frac{1}{7}\operatorname{sen}^7 x + C$

c) $\displaystyle\int \frac{1}{\operatorname{sen}^2 x \cos^2 x} \; dx$ **Sol:** $\tan x - \operatorname{cotan} x + C$

d) $\displaystyle\int \frac{1}{\operatorname{sen}^3 x \cos x} \; dx$ **Sol:** $\displaystyle\ln|\tan x| - \frac{1}{2}\operatorname{cotan}^2 x + C$

3.– Resolver:

a) $\displaystyle\int \frac{1}{\cos x - \operatorname{sen} x} \; dx$ **Sol:** $\displaystyle\frac{1}{\sqrt{2}}\ln\left|\frac{\sqrt{2}+1+\tan\frac{x}{2}}{\sqrt{2}-1-\tan\frac{x}{2}}\right| + C$

b) $\displaystyle\int \frac{5\cos x + 6}{2\cos x + \operatorname{sen} x + 3} \; dx$ **Sol:** $\ln|2\cos x + \operatorname{sen} x + 3| + 2x + C$

c) $\displaystyle\int \frac{1+\tan x}{1-\tan x} \; dx$ **Sol:** $-\ln|\cos x - \operatorname{sen} x| + C$

d) $\displaystyle\int \frac{3\operatorname{sen} x + 2\cos x}{2\operatorname{sen} x + 3\cos x} \; dx$ **Sol:** $\displaystyle-\frac{5}{13}\ln|2\operatorname{sen} x + 3\cos x| + \frac{12}{13}x + C$

4.– Resolver las integrales siguientes:

a) $\displaystyle\int \frac{\operatorname{sen} x}{1+\cos x+\cos 2x} \; dx$ **Sol:** $\ln|2+\sec x| + C$

b) $\displaystyle\int \frac{\tan x}{1+\operatorname{sen}^2 x\tan^2 x} \; dx$ **Sol:** $\displaystyle\frac{1}{\sqrt{3}}\arctan\left(\frac{2\tan^2+1}{\sqrt{3}}\right) + C$

c) $\displaystyle\int \frac{\operatorname{sen} 2x}{(2+\operatorname{sen} x)^2} \; dx$ **Sol:** $\displaystyle\ln(2+\operatorname{sen} x)^2 + \frac{4}{2+\operatorname{sen} x} + C$

d) $\displaystyle\int \frac{1}{\operatorname{sen} x + \operatorname{sen} 2x} \; dx$ **Sol:** $\displaystyle\frac{1}{6}\ln|1-\cos x| + \frac{1}{2}\ln|1+\cos x| - \frac{2}{3}\ln|1+2\cos x| + C$

e) $\displaystyle\int \frac{\cos x}{\cos 2x} \; dx$ **Sol:** $\displaystyle\frac{1}{2\sqrt{2}}\ln\left|\frac{1+\sqrt{2}\operatorname{sen} x}{1-\sqrt{2}\operatorname{sen} x}\right| + C$

f) $\displaystyle\int \frac{1}{\cos x\cos 2x} \; dx$ **Sol:** $\displaystyle\frac{1}{2}\ln\left|\frac{1-\operatorname{sen} x}{1+\operatorname{sen} x}\right| + \frac{1}{\sqrt{2}}\ln\left|\frac{1+\sqrt{2}\operatorname{sen} x}{1-\sqrt{2}\operatorname{sen} x}\right| + C$

g) $\displaystyle\int \frac{\cos^2 x}{4\cos^2 x + \operatorname{sen}^2 x} \; dx$ **Sol:** $\displaystyle\frac{1}{3}\left[x - \frac{1}{2}\arctan\left(\frac{\tan x}{2}\right)\right] + C$

h) $\displaystyle\int \frac{\operatorname{sen} 2x}{1+\operatorname{sen}^2 x} \; dx$ **Sol:** $\ln\left(1+\operatorname{sen}^2 x\right) + C$

5.– Discutir, según los valores del parámetro a, las siguientes integrales:

a) $\displaystyle\int \frac{1}{1+a\cos^2 x}\,dx$

Sol: $a=-1,\ I=-\operatorname{cotan} x+C$

$$a>-1,\ I=\frac{1}{\sqrt{1+a}}\arctan\frac{\tan x}{\sqrt{1+a}}+C$$

$$a<-1,\ I=\frac{-1}{\sqrt{-1-a}}\operatorname{argth}\frac{\tan x}{\sqrt{-1-a}}+C$$

$$I=\frac{-1}{2\sqrt{-1-a}}\ln\left|\frac{\sqrt{-1-a}+\tan x}{\sqrt{-1-a}-\tan x}\right|+C$$

b) $\displaystyle\int \frac{1}{1+a\operatorname{sen}^2 x}\,dx$

Sol: $a=-1,\ I=\tan x+C$

$$a>-1,\ I=\frac{1}{\sqrt{1+a}}\arctan\left(\sqrt{1+a}\tan x\right)+C$$

$$a<-1,\ I=\frac{1}{\sqrt{-1-a}}\operatorname{argth}\left(\sqrt{-1-a}\tan x\right)+C$$

$$I=\frac{1}{2\sqrt{-1-a}}\ln\left|\frac{1+\sqrt{-1-a}\tan x}{1-\sqrt{-1-a}\tan x}\right|+C$$

c) $\displaystyle\int \frac{1}{2-a\operatorname{sen}^2 x}\,dx$

Sol: $a=2,\ I=\dfrac{1}{2}\tan x+C$

$$a<2,\ I=\frac{1}{\sqrt{2}\sqrt{2-a}}\arctan\left(\frac{\sqrt{2-a}\tan x}{\sqrt{2}}\right)+C$$

$$a>2,\ I=\frac{1}{\sqrt{2}\sqrt{a-2}}\operatorname{argth}\left(\frac{\sqrt{a-2}\tan x}{\sqrt{2}}\right)+C$$

$$I=\frac{1}{\sqrt{8}\sqrt{a-2}}\ln\left|\frac{\sqrt{2}+\sqrt{a-2}\tan x}{\sqrt{2}-\sqrt{a-2}\tan x}\right|+C$$

d) $\displaystyle\int \frac{1}{3-a\cos^2 x}\,dx$

Sol: $a=3,\ I=\dfrac{1}{3}\operatorname{cotan} x+C$

$$a<3,\ I=\frac{1}{\sqrt{3}\sqrt{3-a}}\arctan\left(\frac{\sqrt{3}\tan x}{\sqrt{3-a}}\right)+C$$

$$a>3,\ I=\frac{-1}{\sqrt{3}\sqrt{a-3}}\operatorname{argth}\left(\frac{\sqrt{3}\tan x}{\sqrt{a-3}}\right)+C$$

$$I=\frac{-1}{\sqrt{12}\sqrt{a-3}}\ln\left|\frac{\sqrt{a-3}+\sqrt{3}\tan x}{\sqrt{a-3}-\sqrt{3}\tan x}\right|+C$$

6.– Hallar las integrales siguientes:

a) $\displaystyle\int \operatorname{sen}^5 x\,\sqrt[3]{\cos x}\,dx$

Sol: $-3\sqrt[3]{\cos x}\left(\dfrac{\cos^5 x}{16}-\dfrac{\cos^3 x}{5}+\dfrac{\cos x}{4}\right)+C$

b) $\displaystyle\int \frac{1}{\sqrt{\operatorname{sen}^3 x\cos^5 x}}\,dx$

Sol: $\dfrac{2}{3}\sqrt{\tan^3 x}-\dfrac{2}{\sqrt{\tan x}}+C$

c) $\displaystyle\int \operatorname{sen}^3\left(\frac{x}{2}\right)\cos^5\left(\frac{x}{2}\right)\,dx$

Sol: $\dfrac{1}{4}\cos^8\dfrac{x}{2}-\dfrac{1}{3}\cos^6\dfrac{x}{2}+C$

d) $\displaystyle\int \frac{1}{\operatorname{sen}\left(\frac{x}{2}\right)\cos^3\left(\frac{x}{2}\right)}\,dx$

Sol: $2\ln\left|\tan\dfrac{x}{2}\right|+\tan^2\dfrac{x}{2}+C$

e) $\displaystyle\int \operatorname{cotan}^3 x\,\operatorname{cosec}^4 x\,dx$

Sol: $\dfrac{1}{4}\operatorname{cosec}^4 x-\dfrac{1}{6}\operatorname{cosec}^6 x+C$

f) $\displaystyle\int \operatorname{cotan}^3 x\,\operatorname{cosec}^3 x\,dx$

Sol: $\dfrac{1}{3}\operatorname{cosec}^3 x-\dfrac{1}{5}\operatorname{cosec}^5 x+C$
