

1.– Obtain the primitives of the following functions:

a) $\int \frac{3x^2 + 5x - 2}{(x^2 + x)(x^2 + x - 6)} dx$

Sol: $\frac{1}{3} \ln \left| \frac{x(x-2)^2}{(x+1)^2(x+3)} \right| + C$

b) $\int \frac{5x^2 + 42x - 5}{(2x+1)(x^2 + 5x - 6)} dx$

Sol: $\ln \left| \frac{(2x+1)^{3/2}(x-1)^2}{x+6} \right| + C$

c) $\int \frac{2x+1}{x^3 + 5x^2 - x - 5} dx$

Sol: $\frac{1}{8} \left| \frac{(x+1)(x-1)^2}{(x+5)^3} \right| + C$

d) $\int \frac{x^2}{x^3 + 3x^2 + 3x + 1} dx$

Sol: $\ln|x+1| + \frac{4x+3}{2(x+1)^2} + C$

e) $\int \frac{3x+1}{x^2(x+1)^2} dx$

Sol: $\ln \left| \frac{x}{x+1} \right| + \frac{x-1}{x(x+1)} + C$

f) $\int \frac{x^4}{(x^2 - 1)^2} dx$

Sol: $\frac{3}{4} \ln \left| \frac{x-1}{x+1} \right| + \frac{2x^3 - 3x}{2x^3 - 2} + C$

g) $\int \frac{3x^3 - 4x^2 + 3x - 1}{x^3(x-1)^2} dx$

Sol: $\ln \left| \frac{x-1}{x} \right| + \frac{-4x^2 + 3x - 1}{2x^2(x-1)} + C$

h) $\int \frac{x^4 - 6x^3 + 12x^2 + 6}{x^3 - 6x^2 + 12x - 8} dx$

Sol: $\frac{x^2}{2} + \frac{-8x+5}{(x-2)^2} + C$

i) $\int \frac{x-4}{x^3 + 4x^2 + 4x} dx$

Sol: $\ln \left| \frac{x+2}{x} \right| - \frac{3}{x+2} + C$

j) $\int \frac{1}{x^3(1-x^2)} dx$

Sol: $\ln \left| \frac{x}{\sqrt{x^2-1}} \right| - \frac{1}{2x^2} + C$

2.– Integrate the following functions:

a) $\int \frac{3x^2 + 5x + 5}{x^3 + 2x^2 + 5x} dx$

Sol: $\ln|x^3 + 2x^2 + 5x| + \frac{1}{2} \arctan \left(\frac{x+1}{2} \right) + C$

b) $\int \frac{1}{x^3 + 1} dx$

Sol: $\frac{1}{3} \ln \left| \frac{x+1}{\sqrt{x^2-x+1}} \right| + \frac{1}{\sqrt{3}} \arctan \frac{2x-1}{\sqrt{3}} + C$

c) $\int \frac{x^4}{x^4 - 1} dx$

Sol: $x + \frac{1}{4} \ln \left| \frac{x-1}{x+1} \right| - \frac{1}{2} \arctan x + C$

d) $\int \frac{x^3 + x + 1}{x(x^2 + 1)} dx$

Sol: $x + \ln \left| \frac{x}{\sqrt{x^2+1}} \right| + C$

e) $\int \frac{x^2}{(x+1)^2(x^2+1)} dx$

Sol: $\frac{1}{2} \ln \left| \frac{\sqrt{x^2+1}}{x+1} \right| - \frac{1}{2(x+1)} + C$

f) $\int \frac{2x^2 + x + 1}{x^5 + x^3} dx$

Sol: $\ln \left| \frac{x}{\sqrt{x^2+1}} \right| - \arctan x - \frac{2x+1}{2x^2} + C$

g) $\int \frac{1}{x^4 + 1} dx$

Sol: $\frac{\sqrt{2}}{8} \ln \left| \frac{x^2 + \sqrt{2}x + 1}{x^2 - \sqrt{2}x + 1} \right| + \frac{\sqrt{2}}{4} \left(\arctan(\sqrt{2}x+1) + \arctan(\sqrt{2}x-1) \right) + C$

h) $\int \frac{1}{x^4 + x^2 + 1} dx$

Sol: $\frac{1}{4} \ln \left| \frac{x^2 + x + 1}{x^2 - x + 1} \right| + \frac{1}{2\sqrt{3}} \left(\arctan \frac{2x+1}{\sqrt{3}} + \arctan \frac{2x-1}{\sqrt{3}} \right) + C$

3.– Solve the following integrals, depending on the values of the parameters:

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| a) $\int \frac{1}{(x^2 + 2)(x + b)} dx$ | Sol: $\frac{1}{2 + b^2} \left(\ln \left \frac{x + b}{\sqrt{x^2 + 2}} \right + \frac{b}{\sqrt{2}} \arctan \frac{x}{\sqrt{2}} \right) + C$ |
| b) $\int \frac{x}{(x^2 + a^2)(x^2 + 1)} dx$ | Sol: Si $a^2 \neq 1$: $\frac{1}{2(1 - a^2)} \ln \left \frac{x^2 + a^2}{x^2 + 1} \right + C$
Si $a^2 = 1$: $\frac{-1}{2(x^2 + 1)} + C$ |
| c) $\int \frac{1}{(x - a)(x + 1)} dx$ | Sol: Si $a \neq -1$: $\frac{1}{1 + a} \ln \left \frac{x - a}{x + 1} \right + C$
Si $a = -1$: $\frac{-1}{x + 1} + C$ |
| d) $\int \frac{2x}{x^2 + 2bx + c} dx$ | Sol: Si $b \neq 0$: $\frac{1}{b} \ln \left \frac{x}{x + 2b} \right + C$
Si $b = 0$: $-\frac{2}{x} + C$ |
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4.– Solve the following integrals:

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| a) $\int \frac{5x}{(1 - x^2)^8} dx$ | Sol: $\frac{5}{14(1 - x^2)^7} + C$ |
| b) $\int \frac{x^3 + x}{(1 + x^2)^7} dx$ | Sol: $\frac{-1}{10(1 + x^2)^5} + C$ |
| c) $\int \frac{1}{x^4(1 + x^2)} dx$ | Sol: $\frac{-1}{3x^3} + \frac{1}{x} + \arctan x + C$ |
| d) $\int \frac{1}{x^4(x^6 + 1)} dx$ | Sol: $\frac{-1}{3x^3} + -\frac{1}{3} \arctan(x^3) + C$ |
| e) $\int \frac{x^3}{(x^2 - 1)^6} dx$ | Sol: $\frac{-1}{8(x^2 - 1)^4} - \frac{1}{10(x^2 - 1)^5} + C$ |
| f) $\int \frac{x^3}{(1 + x^2)^{10}} dx$ | Sol: $\frac{1}{18(1 + x^2)^9} - \frac{1}{16(1 + x^2)^8} + C$ |
| g) $\int \frac{x^2}{(1 + x^2)^2} dx$ | Sol: $\frac{-x}{2(1 + x^2)} + \frac{1}{2} \arctan x + C$ |
| h) $\int \frac{x^3}{(1 + x^2)^2} dx$ | Sol: $\frac{1}{2(1 + x^2)} + \frac{1}{2} \ln(1 + x^2) + C$ |
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