1.– Obtener las primitivas de las siguientes funciones:

a) $\displaystyle\int \frac{3x^2+5x-2}{(x^2+x)(x^2+x-6)}\,dx$ **Sol:** $\displaystyle\frac{1}{3}\ln\left|\frac{x(x-2)^2}{(x+1)^2(x+3)}\right|+C$

b) $\displaystyle\int \frac{5x^2+42x-5}{(2x+1)(x^2+5x-6)}\,dx$ **Sol:** $\displaystyle\ln\left|\frac{(2x+1)^{3/2}(x-1)^2}{x+6}\right|+C$

c) $\displaystyle\int \frac{2x+1}{x^3+5x^2-x-5}\,dx$ **Sol:** $\displaystyle\frac{1}{8}\left|\frac{(x+1)(x-1)^2}{(x+5)^3}\right|+C$

d) $\displaystyle\int \frac{x^2}{x^3+3x^2+3x+1}\,dx$ **Sol:** $\displaystyle\ln|x+1|+\frac{4x+3}{2(x+1)^2}+C$

e) $\displaystyle\int \frac{3x+1}{x^2(x+1)^2}\,dx$ **Sol:** $\displaystyle\ln\left|\frac{x}{x+1}\right|+\frac{x-1}{x(x+1)}+C$

f) $\displaystyle\int \frac{x^4}{(x^2-1)^2}\,dx$ **Sol:** $\displaystyle\frac{3}{4}\ln\left|\frac{x-1}{x+1}\right|+\frac{2x^3-3x}{2x^3-2}+C$

g) $\displaystyle\int \frac{3x^3-4x^2+3x-1}{x^3(x-1)^2}\,dx$ **Sol:** $\displaystyle\ln\left|\frac{x-1}{x}\right|+\frac{-4x^2+3x-1}{2x^2(x-1)}+C$

h) $\displaystyle\int \frac{x^4-6x^3+12x^2+6}{x^3-6x^2+12x-8}\,dx$ **Sol:** $\displaystyle\frac{x^2}{2}+\frac{-8x+5}{(x-2)^2}+C$

i) $\displaystyle\int \frac{x-4}{x^3+4x^2+4x}\,dx$ **Sol:** $\displaystyle\ln\left|\frac{x+2}{x}\right|-\frac{3}{x+2}+C$

j) $\displaystyle\int \frac{1}{x^3(1-x^2)}\,dx$ **Sol:** $\displaystyle\ln\left|\frac{x}{\sqrt{x^2-1}}\right|-\frac{1}{2x^2}+C$

2.– Calcular las primitivas de las funciones:

a) $\displaystyle\int \frac{3x^2+5x+5}{x^3+2x^2+5x}\,dx$ **Sol:** $\displaystyle\ln\left|x^3+2x^2+5x\right|+\frac{1}{2}\arctan\left(\frac{x+1}{2}\right)+C$

b) $\displaystyle\int \frac{1}{x^3+1}\,dx$ **Sol:** $\displaystyle\frac{1}{3}\ln\left|\frac{x+1}{\sqrt{x^2-x+1}}\right|+\frac{1}{\sqrt{3}}\arctan\frac{2x-1}{\sqrt{3}}+C$

c) $\displaystyle\int \frac{x^4}{x^4-1}\,dx$ **Sol:** $\displaystyle x+\frac{1}{4}\ln\left|\frac{x-1}{x+1}\right|-\frac{1}{2}\arctan x+C$

d) $\displaystyle\int \frac{x^3+x+1}{x(x^2+1)}\,dx$ **Sol:** $\displaystyle x+\ln\left|\frac{x}{\sqrt{x^2+1}}\right|+C$

e) $\displaystyle\int \frac{x^2}{(x+1)^2(x^2+1)}\,dx$ **Sol:** $\displaystyle\frac{1}{2}\ln\left|\frac{\sqrt{x^2+1}}{x+1}\right|-\frac{1}{2(x+1)}+C$

f) $\displaystyle\int \frac{2x^2+x+1}{x^5+x^3}\,dx$ **Sol:** $\displaystyle\ln\left|\frac{x}{\sqrt{x^2+1}}\right|-\arctan x-\frac{2x+1}{2x^2}+C$

g) $\displaystyle\int \frac{1}{x^4+1}\,dx$ **Sol:** $\displaystyle\frac{\sqrt{2}}{8}\ln\left|\frac{x^2+\sqrt{2}x+1}{x^2-\sqrt{2}x+1}\right|+\frac{\sqrt{2}}{4}\Big(\arctan(\sqrt{2}x+1)+\arctan(\sqrt{2}x-1)\Big)+C$

h) $\displaystyle\int \frac{1}{x^4+x^2+1}\,dx$ **Sol:** $\displaystyle\frac{1}{4}\ln\left|\frac{x^2+x+1}{x^2-x+1}\right|+\frac{1}{2\sqrt{3}}\left(\arctan\frac{2x+1}{\sqrt{3}}+\arctan\frac{2x-1}{\sqrt{3}}\right)+C$

3.– Discutir las siguientes integrales, según los valores del parámetro:

a) $\displaystyle\int \frac{1}{(x^2+2)(x+b)}\,dx$ **Sol:** $\displaystyle\frac{1}{2+b^2}\left(\ln\left|\frac{x+b}{\sqrt{x^2+2}}\right| + \frac{b}{\sqrt{2}}\arctan\frac{x}{\sqrt{2}}\right) + C$

b) $\displaystyle\int \frac{x}{(x^2+a^2)(x^2+1)}\,dx$ **Sol:** Si $a^2 \neq 1$: $\displaystyle\frac{1}{2(1-a^2)}\ln\left|\frac{x^2+a^2}{x^2+1}\right| + C$

Si $a^2 = 1$: $\displaystyle\frac{-1}{2(x^2+1)} + C$

c) $\displaystyle\int \frac{1}{(x-a)(x+1)}\,dx$ **Sol:** Si $a \neq -1$: $\displaystyle\frac{1}{1+a}\ln\left|\frac{x-a}{x+1}\right| + C$

Si $a = -1$: $\displaystyle\frac{-1}{x+1} + C$

d) $\displaystyle\int \frac{2x}{x^2+2bx+c}\,dx$ **Sol:** Si $b \neq 0$: $\displaystyle\frac{1}{b}\ln\left|\frac{x}{x+2b}\right| + C$

Si $b = 0$: $\displaystyle -\frac{2}{x} + C$

4.– Calcular las siguientes integrales:

a) $\displaystyle\int \frac{5x}{(1-x^2)^8}\,dx$ **Sol:** $\displaystyle\frac{5}{14(1-x^2)^7} + C$

b) $\displaystyle\int \frac{x^3+x}{(1+x^2)^7}\,dx$ **Sol:** $\displaystyle\frac{-1}{10(1+x^2)^5} + C$

c) $\displaystyle\int \frac{1}{x^4(1+x^2)}\,dx$ **Sol:** $\displaystyle\frac{-1}{3x^3} + \frac{1}{x} + \arctan x + C$

d) $\displaystyle\int \frac{1}{x^4(x^6+1)}\,dx$ **Sol:** $\displaystyle\frac{-1}{3x^3} + -\frac{1}{3}\arctan(x^3) + C$

e) $\displaystyle\int \frac{x^3}{(x^2-1)^6}\,dx$ **Sol:** $\displaystyle\frac{-1}{8(x^2-1)^4} - \frac{1}{10(x^2-1)^5} + C$

f) $\displaystyle\int \frac{x^3}{(1+x^2)^{10}}\,dx$ **Sol:** $\displaystyle\frac{1}{18(1+x^2)^9} - \frac{1}{16(1+x^2)^8} + C$

g) $\displaystyle\int \frac{x^2}{(1+x^2)^2}\,dx$ **Sol:** $\displaystyle\frac{-x}{2(1+x^2)} + \frac{1}{2}\arctan x + C$

h) $\displaystyle\int \frac{x^3}{(1+x^2)^2}\,dx$ **Sol:** $\displaystyle\frac{1}{2(1+x^2)} + \frac{1}{2}\ln(1+x^2) + C$
