4.– Find the reduction formulae for the following integrales:

a) $\displaystyle\int \arcsin^n x \; dx$

Sol: $I(n) = x \arcsin^n x + n\sqrt{1-x^2}\arcsin^{n-1} x - n(n-1)I(n-2)$, valid for $I(0)$ and $I(1)$.

b) $\displaystyle\int \operatorname{argch}^n x \; dx$

Sol: $I(n) = x \operatorname{argch}^n x - n\sqrt{x^2-1}\operatorname{argch}^{n-1} x + n(n-1)I(n-2)$, valid for $I(0)$ and $I(1)$.

c) $\displaystyle\int \tan^n x \; dx$

Sol: $I(n) = \dfrac{1}{n-1}\tan^{n-1} x - I(n-2) \quad (n \neq 1); \quad I(0) = x; \; I(1) = -\ln|\cos x|$

d) $\displaystyle\int \operatorname{cotan}^n x \; dx$

Sol: $I(n) = -\dfrac{1}{n-1}\operatorname{cotan}^{n-1} x - I(n-2) \quad (n \neq 1); \quad I(0) = x; \; I(1) = \ln|\sin x|$

e) $\displaystyle\int \cosh^n x \; dx$

Sol: $I(n) = \dfrac{1}{n}\sinh x \cosh^{n-1} x + \dfrac{n-1}{n} I(n-2) \quad (n \neq 0); \quad I(0) = x; \; I(1) = \sinh x$

f) $\displaystyle\int \left(1+x^2\right)^n \; dx, \; n \in \mathbb{N}$

Sol: $I(n) = \dfrac{x\left(1+x^2\right)^n}{2n+1} + \dfrac{2n}{2n+1} I(n-1)$, valid for $I(0)$.

g) $\displaystyle\int x^n \sinh x \; dx$

Sol: $I(n) = x^n \cosh x - n x^{n-1} \sinh x + n(n-1)I(n-2)$, valid for $I(0)$ and $I(1)$.

h) $\displaystyle\int x^{2n} \cos x \; dx$

Sol: $I(n) = x^{2n} \sin x + 2n\, x^{2n-1} \cos x - 2n(2n-1)\, I(n-1)$, valid for $I(0)$.

i) $\displaystyle\int x^\alpha \ln^n x \; dx, \; \alpha \in \mathbb{R}, \; n \in \mathbb{N}$

Sol: $I(n) = \dfrac{x^{\alpha+1}}{\alpha+1}\ln^n x - \dfrac{n}{\alpha+1} I(n-1) \quad (\alpha \neq -1)$, valid for $I(0)$.

$\alpha = -1: \; I(n) = \dfrac{\ln^{n+1} x}{n+1}$