3.– Solve the following integrales:

a) $\displaystyle\int \frac{x\cos x}{\sin^2 x}\,dx$ **Sol:** $\displaystyle -\frac{x}{\sin x} + \ln\left|\tan\frac{x}{2}\right| + C$

b) $\displaystyle\int \frac{x}{\sin^2 x}\,dx$ **Sol:** $-x\operatorname{cotg} x + \ln|\sin x| + C$

c) $\displaystyle\int \sin\ln x\,dx$ **Sol:** $\displaystyle\frac{x}{2}\left(\sin\ln x - \cos\ln x\right) + C$

d) $\displaystyle\int e^{\sqrt{x}}\,dx$ **Sol:** $2\,e^{\sqrt{x}}\left(\sqrt{x}-1\right) + C$

e) $\displaystyle\int \frac{\arcsin\sqrt{x}}{\sqrt{1-x}}\,dx$ **Sol:** $-2\sqrt{1-x}\,\arcsin\sqrt{x} + 2\sqrt{x} + C$

f) $\displaystyle\int x\ln\left(\frac{1-x}{1+x}\right)\,dx$ **Sol:** $\displaystyle\frac{x^2-1}{2}\ln\left|\frac{1-x}{1+x}\right| - x + C$

g) $\displaystyle\int \cos^2\ln x\,dx$ **Sol:** $\displaystyle\frac{x}{2} + \frac{x\cos(2\ln x)}{10} + \frac{x\sin(2\ln x)}{5} + C$

h) $\displaystyle\int \frac{\sin^2 x}{e^x}\,dx$ **Sol:** $\displaystyle e^{-x}\left(\frac{\cos 2x}{10} - \frac{\sin 2x}{5} - \frac{1}{2}\right) + C$

i) $\displaystyle\int \frac{\arcsin x}{x^2}\,dx$ **Sol:** $\displaystyle -\frac{\arcsin x}{x} + \ln\left|\frac{1-\sqrt{1-x^2}}{x}\right| + C$

j) $\displaystyle\int x\tan^2 2x\,dx$ **Sol:** $\displaystyle\frac{x}{2}\tan 2x - \frac{x^2}{2} + \frac{\ln(\cos 2x)}{4} + C$

k) $\displaystyle\int \arccos\frac{1}{x}\,dx$ **Sol:** $\displaystyle x\arccos\left(\frac{1}{x}\right) - \ln\left|x+\sqrt{x^2-1}\right| + C$

l) $\displaystyle\int \sinh x\ln\left(\cosh^2 x\right)\,dx$ **Sol:** $\cosh x\ln\left(\cosh x\right)^2 - 2\cosh x + C$

4.– Integrate the following functions:

a) $\displaystyle\int e^x \frac{1+\sin x}{1+\cos x}\, dx$ **Sol:** $e^x \tan\frac{x}{2} + C$

b) $\displaystyle\int x^3 \cos x\, dx$ **Sol:** $(x^3 - 6x)\sin x + (3x^2 - 6)\cos x + C$

c) $\displaystyle\int e^x \frac{1+x^2}{(1+x)^2}\, dx$ **Sol:** $\dfrac{x-1}{x+1}\, e^x + C$

d) $\displaystyle\int x^3 \arcsin\frac{1}{x}\, dx$ **Sol:** $\dfrac{1}{4}\left(x^4 \arcsin\dfrac{1}{x} + \dfrac{(x^2-1)^{3/2}}{3} + \sqrt{x^2-1}\right) + C$

e) $\displaystyle\int \left(\frac{\ln x}{x}\right)^2 dx$ **Sol:** $-\dfrac{1}{x}\left(\ln^2 x + 2\ln x + 2\right) + C$

f) $\displaystyle\int \sqrt{x}\, \ln^2 x\, dx$ **Sol:** $\dfrac{2}{3}x^{3/2}\left(\ln^2 x - \dfrac{4}{3}\ln x + \dfrac{8}{9}\right) + C$

g) $\displaystyle\int x^2 \arccos x\, dx$ **Sol:** $-\dfrac{2+x^2}{9}\sqrt{1-x^2} + \dfrac{x^3}{3}\arccos x + C$

h) $\displaystyle\int \arctan\sqrt{x}\, dx$ **Sol:** $(1+x)\arctan\sqrt{x} - \sqrt{x} + C$

i) $\displaystyle\int \sin x\, \ln(\tan x)\, dx$ **Sol:** $-\cos x\, \ln\tan x + \ln\left|\tan\dfrac{x}{2}\right| + C$

j) $\displaystyle\int x\, \arctan^2 x\, dx$ **Sol:** $\dfrac{x^2+1}{2}\arctan^2 x - x\arctan x + \ln\sqrt{1+x^2} + C$

k) $\displaystyle\int \sqrt{x}\, \sin\sqrt{x}\, dx$ **Sol:** $(4-2x)\cos\sqrt{x} + 4\sqrt{x}\sin\sqrt{x} + C$

l) $\displaystyle\int \frac{x\, e^{\arctan x}}{(1+x^2)^{3/2}}\, dx$ **Sol:** $\dfrac{(x-1)\, e^{\arctan x}}{2\sqrt{1+x^2}} + C$