5.– Find the primitives of the following functions:

a) $\displaystyle\int \frac{1}{\sqrt{2x-x^2}}\,dx$ **Sol:** $\arcsin(x-1)+C$ (alternative: $2\arcsin\dfrac{\sqrt{x}}{\sqrt{2}}+C$)

b) $\displaystyle\int \frac{x}{(\alpha^2+x^2)^n}\,dx$ **Sol:** $\dfrac{-1}{2(n-1)(\alpha^2+x^2)^{n-1}}+C$ $(n\neq 1)$; $\dfrac{1}{2}\ln(\alpha^2+x^2)$ $(n=1)$

c) $\displaystyle\int \frac{1}{\cos^4 x}\,dx$ **Sol:** $\tan x+\dfrac{1}{3}\tan^3 x+C$

d) $\displaystyle\int \frac{\cos^2(x/2)}{x+\sin x}\,dx$ **Sol:** $\dfrac{1}{2}\ln|x+\sin x|+C$

e) $\displaystyle\int \frac{\sin x}{(1+\cos^2 x)}\,dx$ **Sol:** $-\arctan(\cos x)+C$

f) $\displaystyle\int \sqrt{1+\sin 2x}\,dx$ **Sol:** $-\sqrt{1-\sin 2x}+C$ (alternative: $\sin x-\cos x+C$)

g) $\displaystyle\int (\cos^4 x-\sin^4 x)\,dx$ **Sol:** $\sin x\,\cos x+C$

h) $\displaystyle\int \sqrt{\frac{1-x}{1+x}}\,dx$ **Sol:** $\arcsin x+\sqrt{1-x^2}+C$

i) $\displaystyle\int \frac{1}{x\sqrt{1-\ln^2 x}}\,dx$ **Sol:** $\arcsin(\ln x)+C$

j) $\displaystyle\int \frac{\cos\ln x}{x}\,dx$ **Sol:** $\sin(\ln x)+C$

k) $\displaystyle\int \sqrt{1+\sin x}\,dx$ **Sol:** $-2\sqrt{1-\sin x}+C$ (alternative: $2\sin\dfrac{x}{2}-2\cos\dfrac{x}{2}+C$)

l) $\displaystyle\int \cos^2 x\,dx$ **Sol:** $\dfrac{x}{2}+\dfrac{1}{4}\sin 2x+C$

m) $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ **Sol:** $2e^{\sqrt{x}}+C$

n) $\displaystyle\int \left(\frac{\sec x}{1+\tan x}\right)^2 dx$ **Sol:** $\dfrac{-1}{1+\tan x}+C$

6.– Obtain the primitives of the following functions:

a) $\displaystyle\int \frac{\sqrt{2+x^2}-\sqrt{2-x^2}}{\sqrt{4-x^4}}\,dx$ **Sol:** $\displaystyle\arcsin\frac{x}{\sqrt{2}} - \ln\left|x+\sqrt{2+x^2}\right| + C$

b) $\displaystyle\int (\tan x + \operatorname{cotan} x)^2\,dx$ **Sol:** $\tan x - \operatorname{cotan} x + C$ (alternative: $-2\operatorname{cotan} 2x + C$)

c) $\displaystyle\int 3^x e^x\,dx$ **Sol:** $\displaystyle\frac{3^x e^x}{1+\ln 3} + C$

d) $\displaystyle\int \frac{\ln x + \sqrt{x}}{x}\,dx$ **Sol:** $\displaystyle\frac{1}{2}\ln^2 x + 2\sqrt{x} + C$

e) $\displaystyle\int \frac{(x^m - x^n)^2}{\sqrt{x}}\,dx$ **Sol:** $\displaystyle\frac{x^{2m+\frac{1}{2}}}{2m+\frac{1}{2}} + \frac{x^{2n+\frac{1}{2}}}{2n+\frac{1}{2}} - \frac{x^{m+n+\frac{1}{2}}}{m+n+\frac{1}{2}} + C$

f) $\displaystyle\int \frac{x^2}{1+x^6}\,dx$ **Sol:** $\displaystyle\frac{1}{3}\arctan x^3 + C$

g) $\displaystyle\int \sqrt{\frac{\arcsin x}{1-x^2}}\,dx$ **Sol:** $\displaystyle\frac{2}{3}(\arcsin x)^{3/2} + C$

h) $\displaystyle\int \frac{\arctan(x/2)}{4+x^2}\,dx$ **Sol:** $\displaystyle\frac{1}{4}\left(\arctan\frac{x}{2}\right)^2 + C$

i) $\displaystyle\int \frac{x-\sqrt{\arctan 2x}}{1+4x^2}\,dx$ **Sol:** $\displaystyle\frac{1}{8}\ln(1+4x^2) - \frac{1}{3}(\arctan 2x)^{3/2} + C$

j) $\displaystyle\int \frac{1}{\sqrt{(1+x^2)\ln(x+\sqrt{1+x^2})}}\,dx$ **Sol:** $\displaystyle 2\sqrt{\ln(x+\sqrt{1+x^2})} + C$

k) $\displaystyle\int \sqrt{\frac{\ln(x+\sqrt{1+x^2})}{1+x^2}}\,dx$ **Sol:** $\displaystyle\frac{2}{3}\left[\ln\left(x+\sqrt{1+x^2}\right)\right]^{3/2} + C$

l) $\displaystyle\int \sinh^2 x\,dx$ **Sol:** $\displaystyle\frac{1}{4}\sinh 2x - \frac{x}{2} + C$

m) $\displaystyle\int \cosh^2 x\,dx$ **Sol:** $\displaystyle\frac{1}{4}\sinh 2x + \frac{x}{2} + C$

n) $\displaystyle\int \frac{\alpha^{2x}-1}{\sqrt{\alpha^x}},\ \alpha > 0\ dx$ **Sol:** $\displaystyle\frac{\alpha^{3x/2}}{3/2\ln\alpha} + \frac{\alpha^{-x/2}}{1/2\ln\alpha} = \frac{2\alpha^{2x}+6}{3\sqrt{\alpha^x}\,\ln\alpha} + C$