# EQUIVALENCES TABLE (FUNCTIONS)

$\lim_{x\to x_0} \alpha(x) = \infty;\qquad \lim_{x\to x_0} \theta(x) = 0;\qquad \lim_{x\to x_0} u(x) = 1;\qquad f_1(x) \sim f_2(x);\qquad g_1(x) \sim g_2(x)$

These equivalences are at $x_0 \begin{cases} x_0 \in \mathbb{R} \\ x_0 = \pm\infty \end{cases}$ ; $f_1(x) \sim f_2(x)$ at $x_0 \Leftrightarrow \lim_{x\to x_0} \dfrac{f_1(x)}{f_2(x)} = 1$

## A. General equivalences

1. $f_1(x) \cdot g_1(x) \sim f_2(x) \cdot g_2(x)$ $\left(\text{If } \exists \lim_{n\to\infty} f_2(x)g_2(x)\right)$
2. $\dfrac{f_1(x)}{g_1(x)} \sim \dfrac{f_2(x)}{g_2(x)}$ $\left(\text{If } \exists \lim_{n\to\infty} \dfrac{f_2(x)}{g_2(x)}\right)$
3. $\log_p(f_1(x)) \sim \log_p(f_2(x))$ $\left(\text{If } \exists \lim_{n\to\infty} f_1(x) \neq 1\right)$

## B. From number $e$

1. $\ln(1+\theta(x)) \sim \theta(x)$
2. $\ln u(x) \sim u(x) - 1$
3. $e^{\theta(x)} - 1 \sim \theta(x)$

Remark: For logarithms on base $p$, we use the relation $\log_p x = \dfrac{\ln x}{\ln p}$

## C. Polynomials

1. $a_0 + a_1\alpha(x) + \ldots + a_p\alpha^p(x) \sim a_p\alpha^p(x)$
2. $\ln(a_0 + a_1\alpha(x) + \ldots + a_p\alpha^p(x)) \sim p\ln\alpha(x)$

## D. Roots

1. $\sqrt[p]{1+\theta(x)} - 1 \sim \dfrac{\theta(x)}{p}$

## E. Trigonometric

1. $\theta(x) \sim \sin\theta(x) \sim \tan\theta(x)$
2. $1 - \cos\theta(x) \sim \dfrac{1}{2}\theta(x)^2$

## F. Change of indetermination

1. $f(x)^{g(x)} \sim e^{g(x)\ln f(x)}$ $\left[\text{for } 1^\infty, 0^0, \infty^0\right]$
2. $\alpha_p(x) - \alpha_q(x) \sim \alpha_p(x)\left(1 - \dfrac{\alpha_q(x)}{\alpha_p(x)}\right)$ $\left[\infty - \infty \to \infty\left(1 - \dfrac{\infty}{\infty}\right)\right]$
3. $\alpha_p(x) - \alpha_q(x) \sim \dfrac{\frac{1}{\alpha_p(x)} - \frac{1}{\alpha_q(x)}}{\frac{1}{\alpha_p(x)\alpha_q(x)}}$ $\left[\infty - \infty \to \dfrac{0}{0}\right]$