Unit IV. Lessons distribution and self-assesment questions.

- Lesson 1. Sections A.1; A.2; A.3.

  1. Can we give any value to the independent variable of a function?
  2. Is there always an order relation between two real functions?
  3. Has the composition of functions the associative property? And the commutative?
  4. Any function must be either odd or even. True or false?

- Lesson 2. Sections B.1 a B.5.

  1. When studying the limit of $f(x)$ at $x = a$, do we need the value of f at a?
  2. If f has a left and a right limit at a point, do they always coincide?
  3. Can a function have a finite limit at infinity? And an infinite limit at a point?
  4. If a function has a limit φ at $x = a$, there is a punctured neighborhood of a in which the function is bounded. True or false?

- Lesson 3. Sections B.6 a B.9.

  1. Let f, g be two functions and $x = a$ a point of their domain. Can we say that the limit of the sum in a is the sum of the limits in a?
  2. What types of indetermination do you know?
  3. In functions, we define infinitesimals for $x = a$ and for $x \to \infty$. True or false?
  4. Obtain the principal part of $x^2 + x$ when $x \to 0$ and when $x \to \infty$.

- Lesson 4. Sections C.1 a C.7.

  1. Can a discontinuity be avoided by defining the value of the function at the point?
  2. Let f and g be continuous. Are their sum, product, and quotient continuous?
  3. Are the functions sine, cosine, and tangent continuous in their fields of existence?
  4. If f is continuous and $f(x_1) \cdot f(x_2) < 0$, then f is zero at some point between x_1 and x_2. True or false?

- Lesson 5. Sections C.8; D.1.1 a D.1.3.

  1. Can a function be uniformly continuous on an open interval?
  2. Can a function be differentiable only from the right?
  3. Can we say that the function $f(x) = 1/x^2$ has a vertical tangent at $x = 0$?
  4. Let f be a differentiable function. What must be true for it to have a derivative?

- Lesson 6. Sections D.1.4 a D.1.6; D.2.

  1. Find a function continuous at a point, but not differentiable at that point.
  2. Find a function differentiable on an interval I, but discontinuous at a point $a \in I$.
  3. If $f \circ g$ is not differentiable, can we assure that one of the functions is not differentiable?
  4. Obtain the derivative of the sine of the sine of x.

- Lesson 7. Sections D.3 a D.5.

  1. Given $f(x) = x^2$, obtain the inverse function $f^{-1}(x)$ and its derivative.
  2. Can Rolle's Theorem be applied to the function $f(x) = 1 - x^{2/3}$ on $[-1, 1]$?
  3. Find a strictly increasing function whose derivative is not positive on all $\mathbb{R}$.
  4. Is the derivative of a continuous function always continuous?

- Lesson 8. Sections D.6 a D.8.1.

  1. By L'Hôpital's rule, the following holds: $\displaystyle \lim_{n\to\infty} \frac{f(x)}{g(x)} = \lim_{n\to\infty} \frac{f'(x)}{g'(x)}$. True or false?
  2. Can the application of L'Hôpital's rule be repeated to obtain a limit?
  3. Does a polynomial of degree k admit a derivative of order higher than k?
  4. Let $P_k(x)$ be the Taylor polynomial of $f(x)$. How many derivatives of $P_k(x)$, coincide with those of $f(x)$ at a?

- Lesson 9. Sections D.8.2 a D.8.6; D.9.

  1. Using Lagrange's remainder term, we can know the exact value of the error made in the approximation. True or false?
  2. Where are the possible extremes of a function?
  3. Say the applications you know of Taylor's expansion.
  4. What is the relation between the Taylor expansion of $f \cdot g$ and those of both functions?