

EQUIVALENCES TABLE (SEQUENCES)

$$\{\alpha_n\} \rightarrow +\infty; \quad \{\theta_n\} \rightarrow 0; \quad \{u_n\} \rightarrow 1; \quad a_n \sim a'_n; \quad b_n \sim b'_n$$

A. General equivalences

$$\begin{array}{lll} 1. & a_n b_n & \sim a'_n b'_n \quad \left(\text{If } \exists \lim_{n \rightarrow \infty} a'_n b'_n \right) \\ 2. & \frac{a_n}{b_n} & \sim \frac{a'_n}{b'_n} \quad \left(\text{If } \exists \lim_{n \rightarrow \infty} \frac{a'_n}{b'_n} \right) \\ 3. & \log_p(a_n) & \sim \log_p(a'_n) \quad \left(\text{If } \exists \lim_{n \rightarrow \infty} a_n \neq 1 \right) \end{array}$$

B. From the number e

$$\begin{array}{lll} 1. & \ln(1 + \theta_n) & \sim \theta_n \\ 2. & \ln u_n & \sim u_n - 1 \\ 3. & e^{\theta_n} - 1 & \sim \theta_n \end{array}$$

Remark: For logarithms of base p , the next relation is used: $\log_p x = \frac{\ln x}{\ln p}$

C. Polynomial expressions

$$\begin{array}{lll} 1. & a_0 + a_1 \alpha_n + \dots + a_p \alpha_n^p & \sim a_p \alpha_n^p \\ 2. & \ln(a_0 + a_1 \alpha_n + \dots + a_p \alpha_n^p) & \sim p \ln \alpha_n \end{array}$$

D. Roots

$$1. \quad \sqrt[p]{1 + \theta_n} - 1 \sim \frac{\theta_n}{p}$$

E. Stirling

$$1. \quad n! \sim n^n e^{-n} \sqrt{2\pi n}$$

F. Trigonometric

$$\begin{array}{lll} 1. & \theta_n & \sim \sin \theta_n \quad \sim \tan \theta_n \\ 2. & 1 - \cos \theta_n & \sim \frac{1}{2} \theta_n^2 \end{array}$$

G. Change of indetermination

$$\begin{array}{lll} 1. & u_n^{\alpha_n} & \sim e^{\alpha_n \ln u_n} \quad [1^\infty \rightarrow e^{\infty 0}] \\ 2. & \theta_n^{\theta'_n} & \sim e^{\theta'_n \ln \theta_n} \quad [0^0 \rightarrow e^{0(-\infty)}] \\ 3. & \alpha_n^{\theta_n} & \sim e^{\theta_n \ln \alpha_n} \quad [\infty^0 \rightarrow e^{0\infty}] \\ 4. & \alpha_n - \alpha'_n & \sim \alpha_n \left(1 - \frac{\alpha'_n}{\alpha_n} \right) \quad \left[\infty - \infty \rightarrow \infty \left(1 - \frac{\infty}{\infty} \right) \right] \end{array}$$