

General term of the sequence of integers (16.07.2022)

There are sequences whose general term has different expressions for even or odd n , for example:

$$\frac{1}{1}, 2, \frac{1}{3}, 4, \dots$$

where a_n is equal to n , for even n and $1/n$, for odd n .

In these cases we may want to obtain a unique expression for the general term. To do this, from the successive powers of -1 , we define the sequences $\{i_n\}$ and $\{p_n\}$:

$$\{(-1)^n, n \in \mathbb{N}\} = -1, 1, -1, 1, \dots \implies \begin{cases} \{i_n\} = \left\{ \frac{1 - (-1)^n}{2} \right\} = 1, 0, 1, 0, 1, 0, \dots \\ \{p_n\} = \left\{ \frac{1 + (-1)^n}{2} \right\} = 0, 1, 0, 1, 0, 1, \dots \end{cases}$$

Now we consider the sequence obtained by rearranging the integers

$$0, 1, -1, 2, -2, 3, -3, \dots$$

The proposed exercise is:

- a) Obtain the expressions of the general term for even and odd n .
- b) Get a single expression for the general term.