1.– In how many ways can all the white pieces in a chess set be placed in a row?

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2.– To set a PIN one must choose four characters from an alphabet of 26 characters. How many PINs can be formed?

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3.– From a collection of 100 paintings, 20 must be chosen for an exhibition. In how many ways can this be done?

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4.– Eight identical sweets are distributed among Ana, Marcos and Fernando. In how many ways can this be done?

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5.– How many numbers can be formed with five digits, all of them different?

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6.– Two professors and ten students stand in a row for a picture. In how many ways can this row be arranged if the teachers must be placed on both ends? What if the teachers can stand anywhere as long as they are separated at least by one student?

(Test, October 2018)

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7.– How many "words" can be formed by rearranging the letters of the word INGENIERO

 (i) in such a way that all vowels appear before the consonants? (for instance OEEIIRNGN is valid, but IEIENOGRN is not)

(ii) in such a way that all consonants are adjacent? (for instance IENGNROEI is valid but ONGRIIEEN is not).

(Final Exam, July 2019)

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8.– An 8-character password is formed by combining letters from the alphabet (assume there are 27) and digits from 0 to 9. Repetitions are allowed.

 (i) How many different passwords can be formed?

 (ii) How many of them contain exactly 6 letters and 2 digits?

(iii) How many of them contain at least one letter and one number?

(Final Exam, July 2020)

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9.–

(i) How many "words" can be formed by rearranging the letters in the word PARALELOS in such a way that no two vowels, neither two consonants, are consecutive?

(ii) Same with the word CANONICA.

(Exam, January 2015)

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10.– A committee of 5 people must be chosen from a group of 11 persons, of which 6 are men and 7 are women.

(i) How many committees can be formed?

(ii) How many if any committee must include at least 2 men and 2 women?

(Final exam, July 2021)

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11.– Consider the alphabet formed by the ten letters A,B,C,D,E,F,G,H,I,J.

(i) How many 3-letter "words" can be formed where all letters are different and placed in alphabetical order?

(ii) How many 8-letter "words" can be formed which contain exactly 3 A's? In how many of them the three A's are consecutive?

(Exam, January 2013)

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12.– 10 marbles are distributed among three girls: Sabela, Lorena and Rita.

(i) In how many ways can this be done if all marbles are different?

(ii) In how many ways if all marbles are identical? And if, moreover, each girl must receive at least two marbles?

(Exam, July 2013)

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13.– Using only numbers 1 and 2

(i) how many different 2×7 matrices can be formed? How many of them have rank 1?

(ii) how many different 2×7 matrices can be formed if each one of them must contain exactly seven 1's and seven 2's?

(Exam, January 2021)

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14.– A word or a number is said to be palindromic if it reads the same backward and forward.

(i) How many 9-digit palindromic numbers are there? In how many of them each digit appears at most two times?

(ii) How many palindromic 21-digit numbers can be formed from exactly eleven 1's and ten 2's?

(Exam, January 2022)

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15.– While re-assembling an electronic device we find five wires which can be connected on any of five spots, but we can't remember what the original configuration looked like.

(i) If we know that each spot takes exactly one wire, what would be the maximum number of trials we would need to do until we find the original configuration?

(ii) If instead we remember that exactly one of the five spots must be left unwired, but we do not know which one, what would be the maximum number of trials?

(Exam, July 2017)

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16.– Let P be a regular polygon with n sides.

(i) How many diagonals does P have?

(ii) How many triangles can be formed by joining vertices from P? How many of them are formed only from diagonals?

(Exam, January 2014)

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17.– If we have 11 friends

(i) in how many ways can we choose five to invite them for lunch?

(ii) Two of them are a couple and they always must go together. In how many ways can we choose five friends under this restriction?

(iii) Two of them are mad at each other and we cannot invite them together. In how many ways can we choose five friends under this restriction?

(Final exam, January 2019)

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I.– Seven people get into an elevator on the ground floor of a building. Each one of them gets off the elevator at one of the remaining five floors.

(i) In how many ways can they get off the elevator, if we take into account who gets off at each floor?

(ii) In how many ways can they get off the elevator, if we only take into account how many people gets off at each floor?.

(Final Exam, July 2015)

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II.– You live in a housing development that can be represented by the following diagram:
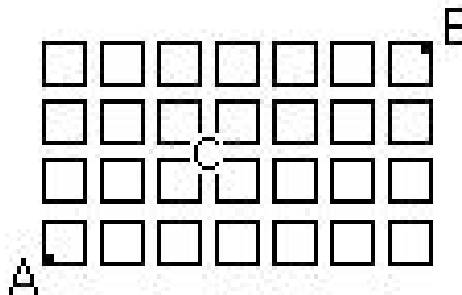
One morning you get ready to go from A to B. Clearly, in order to do this you will have to walk at least 11 block lengths.

(a) How many 11-lengths paths go from A to B?

(b) How many 12-lengths paths?

(c) If you want to avoid the crossing marked with C on the diagram, how many 11-lengths paths you can choose from?

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III.–

i) On the affine plane E_2, what is the maximum number of intersection points determined by 12 different lines?

ii) What if there are at least five lines which are parallel to each other?

iii) What if the lines contain the sides of a regular dodecagon?

(Final Exam, December 2009)

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IV.– Let A be a set with n elements. How many subsets does the set A have? Prove that the number of subsets of A coincide with the number of subsets of A with an odd cardinal.

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V.− Using only 1's and 0's, how many 3×3 matrices can be formed? How many of them have an even trace? How many of them have their determinant equal to 3?

(Partial Exam, October 2015)

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VI.− In a chocolate box there are 3 units of each one of the 5 existing varieties. The chocolates of any given variety are indistinguishable from each other.

(a) Three chocolates are simultaneously taken out of the box. Determine the number of possible configurations.

(b) Four chocolates are simultaneously taken out of the box. Determine the number of possible configurations.

(Final Exam, July 2018)

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VII.− From a collective of 10 men and 8 women, a 12-member committee has to be formed. In how many ways can this be done if

(i) there is no restriction?.

(iii) the committee must include the same number of men and women?

(iii) there must be more men than women in the committee?

(Final Exam, September 2012)

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VIII.− With all digits from 1 to 9

(i) how many numbers of three pairwise different digits can be formed?

(ii) how many of those have an even number as the sum of its digits?

(iii) how many numbers can be formed with five pairwise different digits appearing in decreasing order?.

(Final Exam, January 2017)

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IX.− 13 friends get together at the beach and decide to form 4 teams to play voleyball. Three of the teams will have 3 components and the remaining one will have 4. Among these friends there is a professional player and an unexperienced one. In how many ways can the teams be arranged if the unexperienced player must go in the 4-people team and the professional player must go in one of the 3-people teams?

(Partial Exam, October 2014)

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X.− At a fruit store there are pears, apples, oranges and bananas on sale. They are sold in 6-piece trays, for instance one can buy a tray with 6 apples, or a tray with 3 oranges, 2 pears and an banana.

(a) How many types of tray are there?

(b) How many with at least one piece of each kind?

(Final Exam, 2010)

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