LINEAR ALGEBRA I

Exercises Part II. Chapters 1 and 2

Matrices and determinants.

(Academic year 2022–2023)

Given the matrices:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \\ 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix}, \quad D = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

1.– Calculate $B + D$, AC, CB, $5A$, $2B - 5AC$.

2.– Find B^3.

3.– Calculate A^t, B^t, C^t, D^t.

4.– Calculate $trace(B), trace(D), trace(AC), trace(CA)$.

5.– Decide if any of the matrices A, B, C, D is symmetric or antisymmetric.

6.– Decompose B as the sum of a symmetric matrix and an antisymmetric matrix.

7.– Decide if any of the matrices A, B, C ó D is lower triangular, upper triangular or diagonal.

8.– Compute $det(B), det(D), det(AC), det(CA)$.

9.– Calculate (if it exists) B^{-1}, D^{-1}.

Solutions.

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1. $B + D = \begin{pmatrix} 5 & 2 & 2 \\ 0 & 2 & 2 \\ 1 & 1 & 2 \end{pmatrix}$, $AC = \begin{pmatrix} 3 & 2 & 1 \\ 7 & 4 & 1 \\ 1 & 1 & 1 \end{pmatrix}$, $CB = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 3 & 3 \end{pmatrix}$

$5A = \begin{pmatrix} 5 & 10 \\ 15 & 20 \\ 0 & 5 \end{pmatrix}$, $2B - 5AC = \begin{pmatrix} -13 & -6 & -3 \\ -35 & -18 & -3 \\ -5 & -5 & -3 \end{pmatrix}$.

2. $B^3 = \begin{pmatrix} 1 & 6 & 9 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$.

3. $A^t = \begin{pmatrix} 1 & 3 & 0 \\ 2 & 4 & 1 \end{pmatrix}$, $\quad B^t = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$, $\quad C^t = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{pmatrix}$, $\quad D^t = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

4. $trace(B) = 3, \quad trace(D) = 6, \quad trace(AC) = 8, \quad trace(CA) = 8.$

5. D is symmetric.

6. $B = \begin{pmatrix} 1 & 1 & 1/2 \\ 1 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 1 & 1/2 \\ -1 & 0 & 1/2 \\ -1/2 & -1/2 & 0 \end{pmatrix}$.

7. B is upper triangular.

8. $det(B) = 1, \quad det(D) = -1, \quad det(AC) = 0, \quad det(CA) = 3.$

9. $B^{-1} = \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}, \quad D^{-1} = \begin{pmatrix} 0 & -1 & 1 \\ -1 & -3 & 4 \\ 1 & 4 & -4 \end{pmatrix}.$