LINEAR ALGEBRA I

Exercises Part I, Chapter 1

Sets and functions.

(Academic year 2023–2024)

1.– List the elements of the following sets:

$$A = \{x \in \mathbb{N} | x < 10\},\ B = \{x \in \mathbb{Z} | x^2 < 11\},\ C = \{x \in \mathbb{Q} | 2x \in \mathrm{Z}, -2 < x \leq 1\}.$$

2.– For the sets of the previous exercise calculate:

$$A \cup B,\ A \cap B,\ B \cap C,\ A \cap B \cap C,\ \mathbb{N} \setminus A$$

3.– Ket $A = \{1, 2, 3\}$, $B = \{3, 4, 5, 7\}$ and consider the correspondences:

$$F_1 = \{(1, 3), (2, 7)\}$$
$$F_2 = \{(1, 3), (2, 4), (3, 3)\}$$
$$F_3 = \{(1, 7), (2, 5), (2, 4), (3, 4)\}$$

Represent each of them using a Venn diagram.

4.– For the previous correspondences indicate the initial set, the final set, the domain and the image set.

5.– For the correspondence F_3: What are the images of 2? What are the origins of 4?.

6.– Which of the correspondences in exercise 3 are functions?

7.– For the functions determined in exercise 6, indicate if they are injective, surjective and/or bijective.

8.– Given $f : \mathbb{R} \longrightarrow \mathbb{R}$ defined as $f(x) = 2x + 3$. Indicate if it is bijective. If so, calculate the inverse function.

9.– Given $g : \mathbb{R} \longrightarrow \mathbb{R}$ and $h : \mathbb{R} \longrightarrow \mathbb{R}$ defined respectively as $g(x) = x + 1$ and $h(x) = x^2 - 1$ compute $(g \circ h)$ and $(h \circ g)$.

Solutions.

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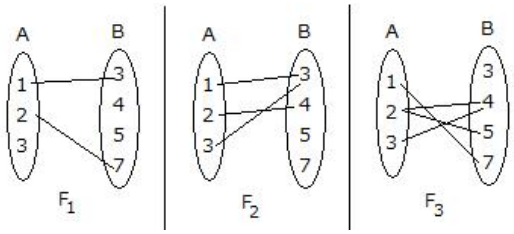
1. $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $B = \{-3, -2, -1, 0, 1, 2, 3\}$, $C = \{-3/2, -1, -1/2, 0, 1/2, 1\}$.

2. $A \cup B = \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, $A \cap B = \{1, 2, 3\}$, $B \cap C = \{-1, 0, 1\}$.

$A \cap B \cap C = \{1\}$, $\mathbb{N} \setminus A = \{x \in \mathbb{N} | x \geq 10\}$.

3.



4. For all correspondences the initial set is $A = \{1, 2, 3\}$ and the final set is $B = \{3, 4, 5, 7\}$.

$Dominio(F_1) = \{1, 2\}, \quad C.Imagen(F_1) = \{3, 7\}.$

$Dominio(F_2) = \{1, 2, 3\}, \quad C.Imagen(F_2) = \{3, 4\}.$

$Dominio(F_3) = \{1, 2, 3\}, \quad C.Imagen(F_3) = \{4, 5, 7\}.$

5. The images of the 2 are $4, 5$. The origins of the 4 are $2, 3$.

6. Only F_2 is functions.

7. F_2 is neither injective, nor surjective, nor bijective.

8. It is bijective. $f^{-1}(x) = \dfrac{x-3}{2}$.

9. $(g \circ h)(x) = x^2, \quad (h \circ g)(x) = x^2 + 2x.$