LINEAR ALGEBRA I

Exercises Part I, Chapter 1

Sets and functions.

(Academic year 2022–2023)

1.— Write down the elements of the following sets:

$$A = \{x \in \mathbb{N} | x < 10\}, B = \{x \in \mathbb{Z} | x^2 < 11\}, C = \{x \in \mathbb{Q} | 2x \in \mathbb{Z}, -2 < x \le 1\}.$$

2.— For the sets defined in the previous exercise calculate:

$$A \cup B$$
, $A \cap B$, $B \cap C$, $A \cap B \cap C$, $\mathbb{N} \setminus A$

3.— Put $A = \{1, 2, 3\}, B = \{3, 4, 5, 7\}$ and consider the correspondences:

$$F_1 = \{(1,3), (2,7)\}$$

$$F_2 = \{(1,3), (2,4), (3,3)\}$$

$$F_3 = \{(1,7), (2,5), (2,4), (3,4)\}$$

Represent each of them using a Venn diagram.

- 4.— For the previous correspondences indicate the initial set, the final set, the domain and the image set.
- **5.** For the correspondence F_3 : What are the images of 2? What are the origins of 4?.
- **6.** Which of the correspondences in exercise 3 are functions?
- 7.— For the functions determined in exercise 6, indicate if they are injective, surjective and/or bijective.
- **8.** Given $f: \mathbb{R} \longrightarrow \mathbb{R}$ defined as f(x) = 2x + 3. Indicate if it is bijective. If so, calculate the inverse function.
- **9.** Given $g: \mathbb{R} \longrightarrow \mathbb{R}$ and $h: \mathbb{R} \longrightarrow \mathbb{R}$ defined respectively as g(x) = x + 1 and $h(x) = x^2 1$ compute $(g \circ h)$ and $(h \circ g)$.

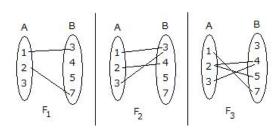
Solutions.

1. $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}, \quad B = \{-3, -2, -1, 0, 1, 2, 3\}, \quad C = \{-3/2, -1, -1/2, 0, 1/2, 1\}.$

2.
$$A \cup B = \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}, A \cap B = \{1, 2, 3\}, B \cap C = \{-1, 0, 1\}.$$

 $A \cap B \cap C = \{1\}, \quad \mathbb{N} \setminus A = \{x \in \mathbb{N} | x \ge 10\}.$

3.



4. For all correspondences the initial set is $A = \{1, 2, 3\}$ and the final set is $B = \{3, 4, 5, 7\}$.

Domain $(F_1) = \{1, 2\}, \text{ Image set}(F_1) = \{3, 7\}.$

Domain $(F_2) = \{1, 2, 3\}, \text{ Image set}(F_2) = \{3, 4\}.$

Domain = $\{1, 2, 3\}$, Image set $(F_3) = \{4, 5, 7\}$.

5. The images of the 2 are 4, 5. The origins of the 4 are 2, 3.

6. Only F_2 is a function.

7. F_2 is neither injective, nor surjective, nor bijective.

- **8.** It is bijective. $f^{-1}(x) = \frac{x-3}{2}$.
- **9.** $(g \circ h)(x) = x^2$, $(h \circ g)(x) = x^2 + 2x$.