1. MÉTODOS DE INTERVALO SIMPLE

1.1. Métodos basados en la aproximación de la derivada

1.1.1. Método de Euler

$$\boldsymbol{y}_{i+1} = \boldsymbol{y}_i + \Delta t \; \boldsymbol{\varphi}(t_i, \boldsymbol{y}_i) \qquad ; \tau(\Delta t)$$

1.1.2 Método de Diferencias Centradas

$$\boldsymbol{y}_{i+1} = \boldsymbol{y}_{i-1} + 2\Delta t \; \boldsymbol{\varphi}(t_i, \boldsymbol{y}_i) \qquad ; \tau(\Delta t^2)$$

1.2. Métodos basados en desarrollos en serie

1.2.1. Método del Desarrollo en Serie de Segundo Orden

$$\boldsymbol{y}_{i+1} = \boldsymbol{y}_i + \Delta t \; \boldsymbol{\varphi}(t_i, \boldsymbol{y}_i) + \frac{\Delta t^2}{2} \Big(\boldsymbol{\varphi}'_t(t_i, \boldsymbol{y}_i) + \boldsymbol{\varphi}'_{\boldsymbol{y}}(t_i, \boldsymbol{y}_i) \boldsymbol{\varphi}(t_i, \boldsymbol{y}_i) \Big) \qquad ; \tau(\Delta t^2)$$

1.3. Métodos de Runge-Kutta

1.3.1. Métodos de Runge-Kutta de Segundo Orden

$$\begin{aligned} \boldsymbol{y}_{i+1} = \boldsymbol{y}_i + \Delta t \; \boldsymbol{\Phi}(t_i, \boldsymbol{y}_i) & \\ \boldsymbol{\Phi}(t, \boldsymbol{y}) =& w_0 \boldsymbol{k}_0 + w_1 \boldsymbol{k}_1 \\ \boldsymbol{k}_0 =& \boldsymbol{\varphi}(t, \boldsymbol{y}) \\ \boldsymbol{k}_1 =& \boldsymbol{\varphi}(t + \theta_1 \Delta t, \boldsymbol{y} + (w_{10} \boldsymbol{k}_0) \Delta t) \end{aligned}$$

$$w_0 + w_1 = 1; \qquad w_1 \theta_1 = \frac{1}{2}; \qquad w_1 w_{10} = \frac{1}{2}$$

1.3.1.1. Método de Euler Modificado

$$w_0 = 0; \qquad w_1 = 1; \qquad \theta_1 = \frac{1}{2}; \qquad w_{10} = \frac{1}{2} \qquad ; \tau(\Delta t^2)$$

1.3.1.2. Método de Heun

$$w_0 = \frac{1}{2}; \qquad w_1 = \frac{1}{2}; \qquad \theta_1 = 1; \qquad w_{10} = 1 \qquad ; \tau(\Delta t^2)$$

1.3.1.3. Método de Ralston

$$w_0 = \frac{1}{3}; \qquad w_1 = \frac{2}{3}; \qquad \theta_1 = \frac{3}{4}; \qquad w_{10} = \frac{3}{4} \qquad ; \tau(\Delta t^2)$$

1.3.1.4. Método de Tercer Orden para $\boldsymbol{\varphi}'_{\boldsymbol{y}} = 0$

$$w_0 = \frac{1}{4}; \qquad w_1 = \frac{3}{4}; \qquad \theta_1 = \frac{2}{3}; \qquad w_{10} = \frac{2}{3} \qquad ; \tau(\Delta t^2)$$

1.3.2. Métodos de Runge-Kutta de Tercer Orden

$$\begin{aligned}
\boldsymbol{y}_{i+1} &= \boldsymbol{y}_i + \Delta t\ \boldsymbol{\Phi}(t_i, \boldsymbol{y}_i) \\
\boldsymbol{\Phi}(t, \boldsymbol{y}) &= w_0 \boldsymbol{k}_0 + w_1 \boldsymbol{k}_1 + w_2 \boldsymbol{k}_2 \\
\boldsymbol{k}_0 &= \boldsymbol{\varphi}(t, \boldsymbol{y}) \\
\boldsymbol{k}_1 &= \boldsymbol{\varphi}(t + \theta_1 \Delta t, \boldsymbol{y} + (w_{10} \boldsymbol{k}_0) \Delta t) \\
\boldsymbol{k}_2 &= \boldsymbol{\varphi}(t + \theta_2 \Delta t, \boldsymbol{y} + (w_{20} \boldsymbol{k}_0 + w_{21} \boldsymbol{k}_1) \Delta t)
\end{aligned}$$

$$\begin{aligned}
w_0 + w_1 + w_2 &= 1; & w_1\theta_1 + w_2\theta_2 &= \frac{1}{2}; & w_1\theta_1^2 + w_2\theta_2^2 &= \frac{1}{3} \\
w_2\theta_1 w_{21} &= \frac{1}{6}; & \theta_1 &= w_{10}; & \theta_2 &= w_{20} + w_{21}
\end{aligned}$$

1.3.2.1. Método de Kutta

$$\begin{aligned}
w_0 &= \frac{1}{6}; & w_1 &= \frac{4}{6}; & w_2 &= \frac{1}{6} \\
\theta_1 &= \frac{1}{2}; & \theta_2 &= 1; & & \\
w_{10} &= \frac{1}{2}; & w_{20} &= -1; & w_{21} &= 2
\end{aligned} \qquad ; \tau(\Delta t^3)$$

1.3.2.2. Método de Heun de Tercer Orden

$$\begin{aligned}
w_0 &= \frac{1}{4}; & w_1 &= 0; & w_2 &= \frac{3}{4} \\
\theta_1 &= \frac{1}{3}; & \theta_2 &= \frac{2}{3}; & & \\
w_{10} &= \frac{1}{3}; & w_{20} &= 0; & w_{21} &= \frac{2}{3}
\end{aligned} \qquad ; \tau(\Delta t^3)$$

1.3.3. Métodos de Runge-Kutta de Cuarto Orden

$$
\begin{aligned}
\boldsymbol{y}_{i+1} = \boldsymbol{y}_i + \Delta t\ \boldsymbol{\Phi}(t_i, \boldsymbol{y}_i)\\
\boldsymbol{\Phi}(t,\boldsymbol{y}) &= w_0\boldsymbol{k}_0 + w_1\boldsymbol{k}_1 + w_2\boldsymbol{k}_2 + w_3\boldsymbol{k}_3\\
\boldsymbol{k}_0 &= \boldsymbol{\varphi}(t,\boldsymbol{y})\\
\boldsymbol{k}_1 &= \boldsymbol{\varphi}(t+\theta_1\Delta t, \boldsymbol{y} + (w_{10}\boldsymbol{k}_0)\Delta t)\\
\boldsymbol{k}_2 &= \boldsymbol{\varphi}(t+\theta_2\Delta t, \boldsymbol{y} + (w_{20}\boldsymbol{k}_0 + w_{21}\boldsymbol{k}_1)\Delta t)\\
\boldsymbol{k}_3 &= \boldsymbol{\varphi}(t+\theta_3\Delta t, \boldsymbol{y} + (w_{30}\boldsymbol{k}_0 + w_{31}\boldsymbol{k}_1 + w_{32}\boldsymbol{k}_2)\Delta t)
\end{aligned}
$$

1.3.3.1. Método de Kutta de Cuarto Orden

$$
\begin{aligned}
w_0 &= \frac{1}{6}; & w_1 &= \frac{1}{3}; & w_2 &= \frac{1}{3}; & w_3 &= \frac{1}{6}\\
\theta_1 &= \frac{1}{2}; & \theta_2 &= \frac{1}{2}; & \theta_3 &= 1\\
w_{10} &= \frac{1}{2}; & w_{20} &= 0; & w_{21} &= \frac{1}{2}\\
w_{30} &= 0; & w_{31} &= 0; & w_{32} &= 1
\end{aligned}
\quad ; \tau(\Delta t^4)
$$

1.3.3.2. Método de cuarto orden asociado a la cuadratura de Newton-Cotes

$$
\begin{aligned}
w_0 &= \frac{1}{8}; & w_1 &= \frac{3}{8}; & w_2 &= \frac{3}{8}; & w_3 &= \frac{1}{8}\\
\theta_1 &= \frac{1}{3}; & \theta_2 &= \frac{2}{3}; & \theta_3 &= 1\\
w_{10} &= \frac{1}{3}; & w_{20} &= -\frac{1}{3}; & w_{21} &= 1\\
w_{30} &= 1; & w_{31} &= -1; & w_{32} &= 1
\end{aligned}
\quad ; \tau(\Delta t^4)
$$

1.3.3.3. Método de Gill

$$
\begin{aligned}
w_0 &= \frac{1}{6}; & w_1 &= \frac{2}{6}(1-\frac{1}{\sqrt{2}}); & w_2 &= \frac{2}{6}(1+\frac{1}{\sqrt{2}}); & w_3 &= \frac{1}{6}\\
\theta_1 &= \frac{1}{2}; & \theta_2 &= \frac{1}{2}; & \theta_3 &= 1\\
w_{10} &= \frac{1}{2}; & w_{20} &= (-\frac{1}{2}+\frac{1}{\sqrt{2}}); & w_{21} &= (1-\frac{1}{\sqrt{2}})\\
w_{30} &= 0; & w_{31} &= -\frac{1}{\sqrt{2}}; & w_{32} &= (1+\frac{1}{\sqrt{2}})
\end{aligned}
\quad ; \tau(\Delta t^4)
$$

2. MÉTODOS DE INTERVALO MULTIPLE

2.1. Fórmulas Abiertas (PREDICTORES) (Métodos de Adams-Bashford)

2.1.1. k=0, r=3

$$\boldsymbol{y}_{i+1} = \boldsymbol{y}_i + \frac{\Delta t}{24}(55\boldsymbol{\varphi}_i - 59\boldsymbol{\varphi}_{i-1} + 37\boldsymbol{\varphi}_{i-2} - 9\boldsymbol{\varphi}_{i-3}) \qquad ;\tau(\Delta t^4)$$

2.1.2. k=1, r=1

$$\boldsymbol{y}_{i+1} = \boldsymbol{y}_{i-1} + 2\Delta t\boldsymbol{\varphi}_i \qquad ;\tau(\Delta t^2)$$

2.1.3. k=3, r=3

$$\boldsymbol{y}_{i+1} = \boldsymbol{y}_{i-3} + \frac{4\Delta t}{3}(2\boldsymbol{\varphi}_i - \boldsymbol{\varphi}_{i-1} + 2\boldsymbol{\varphi}_{i-2}) \qquad ;\tau(\Delta t^4)$$

2.1.4. k=5, r=5

$$\boldsymbol{y}_{i+1} = \boldsymbol{y}_{i-5} + \frac{3\Delta t}{10}(11\boldsymbol{\varphi}_i - 14\boldsymbol{\varphi}_{i-1} + 26\boldsymbol{\varphi}_{i-2} - 14\boldsymbol{\varphi}_{i-3} + 11\boldsymbol{\varphi}_{i-4}) \qquad ;\tau(\Delta t^6)$$

2.2. Fórmulas Cerradas (CORRECTORES) (Métodos de Moulton)

2.2.1. k=0, r=3

$$\boldsymbol{y}_{i+1} = \boldsymbol{y}_i + \frac{\Delta t}{24}(9\boldsymbol{\varphi}_{i+1} + 19\boldsymbol{\varphi}_i - 5\boldsymbol{\varphi}_{i-1} + \boldsymbol{\varphi}_{i-2}) \qquad ;\tau(\Delta t^4)$$

2.2.2. k=1, r=3

$$\boldsymbol{y}_{i+1} = \boldsymbol{y}_{i-1} + \frac{\Delta t}{3}(\boldsymbol{\varphi}_{i+1} + 4\boldsymbol{\varphi}_i + \boldsymbol{\varphi}_{i-1}) \qquad ;\tau(\Delta t^4)$$

2.2.3. k=3, r=5

$$\boldsymbol{y}_{i+1} = \boldsymbol{y}_{i-3} + \frac{2\Delta t}{45}(7\boldsymbol{\varphi}_{i+1} + 32\boldsymbol{\varphi}_i + 12\boldsymbol{\varphi}_{i-1} + 32\boldsymbol{\varphi}_{i-2} + 7\boldsymbol{\varphi}_{i-3}) \qquad ;\tau(\Delta t^6)$$

2.3. Métodos PREDICTOR-CORRECTOR

2.3.1. Método de Adams-Moulton de Cuarto Orden

Predictor: k=0, r=3
Corrector: k=0, r=3

2.3.2. Método de Milne de Cuarto Orden

Predictor: k=3, r=3
Corrector: k=1, r=3

2.3.3. Método de Milne de Sexto Orden

Predictor: k=5, r=5
Corrector: k=3, r=5