1. MÉTODOS DE INTERVALO SIMPLE

1.1. Métodos basados en la aproximación de la derivada

1.1.1. Método de Euler

$$\boldsymbol{y}_{i+1} = \boldsymbol{y}_i + \Delta t\ \boldsymbol{\varphi}(t_i, \boldsymbol{y}_i) \qquad ; \tau(\Delta t)$$

1.1.2 Método de Diferencias Centradas

$$\boldsymbol{y}_{i+1} = \boldsymbol{y}_{i-1} + 2\Delta t\ \boldsymbol{\varphi}(t_i, \boldsymbol{y}_i) \qquad ; \tau(\Delta t^2)$$

1.2. Métodos basados en desarrollos en serie

1.2.1. Método del Desarrollo en Serie de Segundo Orden

$$\boldsymbol{y}_{i+1} = \boldsymbol{y}_i + \Delta t\ \boldsymbol{\varphi}(t_i, \boldsymbol{y}_i) + \frac{\Delta t^2}{2}\Big(\boldsymbol{\varphi}'_t(t_i, \boldsymbol{y}_i) + \boldsymbol{\varphi}'_{\boldsymbol{y}}(t_i, \boldsymbol{y}_i)\boldsymbol{\varphi}(t_i, \boldsymbol{y}_i)\Big) \qquad ; \tau(\Delta t^2)$$

1.3. Métodos de Runge-Kutta

1.3.1. Métodos de Runge-Kutta de Segundo Orden

$$\begin{aligned} \boldsymbol{y}_{i+1} = \boldsymbol{y}_i + \Delta t\ &\boldsymbol{\Phi}(t_i, \boldsymbol{y}_i) \\ \boldsymbol{\Phi}(t, \boldsymbol{y}) =& w_0\boldsymbol{k}_0 + w_1\boldsymbol{k}_1 \\ \boldsymbol{k}_0 =& \boldsymbol{\varphi}(t, \boldsymbol{y}) \\ \boldsymbol{k}_1 =& \boldsymbol{\varphi}(t + \theta_1\Delta t, \boldsymbol{y} + (w_{10}\boldsymbol{k}_0)\Delta t) \end{aligned}$$

$$w_0 + w_1 = 1; \qquad w_1\theta_1 = \frac{1}{2}; \qquad w_1 w_{10} = \frac{1}{2}$$

1.3.1.1. Método de Euler Modificado

$$w_0 = 0; \qquad w_1 = 1; \qquad \theta_1 = \frac{1}{2}; \qquad w_{10} = \frac{1}{2} \qquad ; \tau(\Delta t^2)$$

1.3.1.2. Método de Heun

$$w_0 = \frac{1}{2}; \qquad w_1 = \frac{1}{2}; \qquad \theta_1 = 1; \qquad w_{10} = 1 \qquad ; \tau(\Delta t^2)$$

1.3.1.3. Método de Ralston

$$w_0 = \frac{1}{3}; \qquad w_1 = \frac{2}{3}; \qquad \theta_1 = \frac{3}{4}; \qquad w_{10} = \frac{3}{4} \qquad ; \tau(\Delta t^2)$$

1.3.1.4. Método de Tercer Orden para $\boldsymbol{\varphi}'_{\boldsymbol{y}} = 0$

$$w_0 = \frac{1}{4}; \qquad w_1 = \frac{3}{4}; \qquad \theta_1 = \frac{2}{3}; \qquad w_{10} = \frac{2}{3} \qquad ; \tau(\Delta t^2)$$

1.3.2. Métodos de Runge-Kutta de Tercer Orden

$$\begin{aligned}
\boldsymbol{y}_{i+1} = \boldsymbol{y}_i + \Delta t\ \boldsymbol{\Phi}(t_i, \boldsymbol{y}_i)\\
\boldsymbol{\Phi}(t, \boldsymbol{y}) =& w_0 \boldsymbol{k}_0 + w_1 \boldsymbol{k}_1 + w_2 \boldsymbol{k}_2\\
\boldsymbol{k}_0 =& \boldsymbol{\varphi}(t, \boldsymbol{y})\\
\boldsymbol{k}_1 =& \boldsymbol{\varphi}(t + \theta_1 \Delta t, \boldsymbol{y} + (w_{10} \boldsymbol{k}_0) \Delta t)\\
\boldsymbol{k}_2 =& \boldsymbol{\varphi}(t + \theta_2 \Delta t, \boldsymbol{y} + (w_{20} \boldsymbol{k}_0 + w_{21} \boldsymbol{k}_1) \Delta t)
\end{aligned}$$

$$\begin{aligned}
w_0 + w_1 + w_2 =& 1; & w_1 \theta_1 + w_2 \theta_2 =& \frac{1}{2}; & w_1 \theta_1^2 + w_2 \theta_2^2 =& \frac{1}{3}\\
w_2 \theta_1 w_{21} =& \frac{1}{6}; & \theta_1 =& w_{10}; & \theta_2 =& w_{20} + w_{21}
\end{aligned}$$

1.3.2.1. Método de Kutta

$$\begin{aligned}
w_0 =& \frac{1}{6}; & w_1 =& \frac{4}{6}; & w_2 =& \frac{1}{6}\\
\theta_1 =& \frac{1}{2}; & \theta_2 =& 1; & &\\
w_{10} =& \frac{1}{2}; & w_{20} =& -1; & w_{21} =& 2
\end{aligned} \qquad ; \tau(\Delta t^3)$$

1.3.2.2. Método de Heun de Tercer Orden

$$\begin{aligned}
w_0 =& \frac{1}{4}; & w_1 =& 0; & w_2 =& \frac{3}{4}\\
\theta_1 =& \frac{1}{3}; & \theta_2 =& \frac{2}{3}; & &\\
w_{10} =& \frac{1}{3}; & w_{20} =& 0; & w_{21} =& \frac{2}{3}
\end{aligned} \qquad ; \tau(\Delta t^3)$$

1.3.3. Métodos de Runge-Kutta de Cuarto Orden

$$
\begin{aligned}
\boldsymbol{y}_{i+1} = \boldsymbol{y}_i + \Delta t \; \boldsymbol{\Phi}(t_i, \boldsymbol{y}_i) \\
\boldsymbol{\Phi}(t, \boldsymbol{y}) =& w_0 \boldsymbol{k}_0 + w_1 \boldsymbol{k}_1 + w_2 \boldsymbol{k}_2 + w_3 \boldsymbol{k}_3 \\
\boldsymbol{k}_0 =& \boldsymbol{\varphi}(t, \boldsymbol{y}) \\
\boldsymbol{k}_1 =& \boldsymbol{\varphi}(t + \theta_1 \Delta t, \boldsymbol{y} + (w_{10} \boldsymbol{k}_0) \Delta t) \\
\boldsymbol{k}_2 =& \boldsymbol{\varphi}(t + \theta_2 \Delta t, \boldsymbol{y} + (w_{20} \boldsymbol{k}_0 + w_{21} \boldsymbol{k}_1) \Delta t) \\
\boldsymbol{k}_3 =& \boldsymbol{\varphi}(t + \theta_3 \Delta t, \boldsymbol{y} + (w_{30} \boldsymbol{k}_0 + w_{31} \boldsymbol{k}_1 + w_{32} \boldsymbol{k}_2) \Delta t)
\end{aligned}
$$

1.3.3.1. Método de Kutta de Cuarto Orden

$$
\begin{matrix}
w_0 = \frac{1}{6}; & w_1 = \frac{1}{3}; & w_2 = \frac{1}{3}; & w_3 = \frac{1}{6} \\
\theta_1 = \frac{1}{2}; & \theta_2 = \frac{1}{2}; & \theta_3 = 1 & \\
w_{10} = \frac{1}{2}; & w_{20} = 0; & w_{21} = \frac{1}{2} & \\
w_{30} = 0; & w_{31} = 0; & w_{32} = 1 &
\end{matrix}
\quad ; \tau(\Delta t^4)
$$

1.3.3.2. Método de cuarto orden asociado a la cuadratura de Newton-Cotes

$$
\begin{matrix}
w_0 = \frac{1}{8}; & w_1 = \frac{3}{8}; & w_2 = \frac{3}{8}; & w_3 = \frac{1}{8} \\
\theta_1 = \frac{1}{3}; & \theta_2 = \frac{2}{3}; & \theta_3 = 1 & \\
w_{10} = \frac{1}{3}; & w_{20} = -\frac{1}{3}; & w_{21} = 1 & \\
w_{30} = 1; & w_{31} = -1; & w_{32} = 1 &
\end{matrix}
\quad ; \tau(\Delta t^4)
$$

1.3.3.3. Método de Gill

$$
\begin{matrix}
w_0 = \frac{1}{6}; & w_1 = \frac{2}{6}(1 - \frac{1}{\sqrt{2}}); & w_2 = \frac{2}{6}(1 + \frac{1}{\sqrt{2}}); & w_3 = \frac{1}{6} \\
\theta_1 = \frac{1}{2}; & \theta_2 = \frac{1}{2}; & \theta_3 = 1 & \\
w_{10} = \frac{1}{2}; & w_{20} = (-\frac{1}{2} + \frac{1}{\sqrt{2}}); & w_{21} = (1 - \frac{1}{\sqrt{2}}) & \\
w_{30} = 0; & w_{31} = -\frac{1}{\sqrt{2}}; & w_{32} = (1 + \frac{1}{\sqrt{2}}) &
\end{matrix}
\quad ; \tau(\Delta t^4)
$$