

1. MÉTODOS DE INTERVALO SIMPLE

1.1. Métodos basados en la aproximación de la derivada

1.1.1. Método de Euler

$$y_{i+1} = y_i + h\varphi(x_i, y_i) \quad ; \tau(h)$$

1.1.2 Método de Diferencias Centradas

$$y_{i+1} = y_{i-1} + 2h\varphi(x_i, y_i) \quad ; \tau(h^2)$$

1.2. Métodos basados en desarrollos en serie

1.2.1. Método del Desarrollo en Serie de Segundo Orden

$$y_{i+1} = y_i + h\varphi(x_i, y_i) + \frac{h^2}{2} (\varphi'_x(x_i, y_i) + \varphi'_y(x_i, y_i)\varphi(x_i, y_i)) \quad ; \tau(h^2)$$

1.3. Métodos de Runge-Kutta

1.3.1. Métodos de Runge-Kutta de Segundo Orden

$$\begin{aligned} y_{i+1} &= y_i + h\Phi(x_i, y_i) \\ \Phi(x, y) &= w_0k_0 + w_1k_1 \\ k_0 &= \varphi(x, y) \\ k_1 &= \varphi(x + \theta_1h, y + (w_{10}k_0)h) \end{aligned}$$

$$w_0 + w_1 = 1; \quad w_1\theta_1 = \frac{1}{2}; \quad w_1w_{10} = \frac{1}{2}$$

1.3.1.1. Método de Euler Modificado

$$w_0 = 0; \quad w_1 = 1; \quad \theta_1 = \frac{1}{2}; \quad w_{10} = \frac{1}{2} \quad ; \tau(h^2)$$

1.3.1.2. Método de Heun

$$w_0 = \frac{1}{2}; \quad w_1 = \frac{1}{2}; \quad \theta_1 = 1; \quad w_{10} = 1 \quad ; \tau(h^2)$$

1.3.1.3. Método de Ralston

$$w_0 = \frac{1}{3}; \quad w_1 = \frac{2}{3}; \quad \theta_1 = \frac{3}{4}; \quad w_{10} = \frac{3}{4} \quad ; \tau(h^2)$$

1.3.1.4. Método de Tercer Orden para $\varphi'_y = 0$

$$w_0 = \frac{1}{4}; \quad w_1 = \frac{3}{4}; \quad \theta_1 = \frac{2}{3}; \quad w_{10} = \frac{2}{3} \quad ; \tau(h^2)$$

1.3.2. Métodos de Runge-Kutta de Tercer Orden

$$y_{i+1} = y_i + h\Phi(x_i, y_i)$$

$$\Phi(x, y) = w_0k_0 + w_1k_1 + w_2k_2$$

$$k_0 = \varphi(x, y)$$

$$k_1 = \varphi(x + \theta_1h, y + (w_{10}k_0)h)$$

$$k_2 = \varphi(x + \theta_2h, y + (w_{20}k_0 + w_{21}k_1)h)$$

$$\begin{aligned} w_0 + w_1 + w_2 &= 1; & w_1\theta_1 + w_2\theta_2 &= \frac{1}{2}; & w_1\theta_1^2 + w_2\theta_2^2 &= \frac{1}{3} \\ w_2\theta_1w_{21} &= \frac{1}{6}; & \theta_1 &= w_{10}; & \theta_2 &= w_{20} + w_{21} \end{aligned}$$

1.3.2.1. Método de Kutta

$$\begin{aligned} w_0 &= \frac{1}{6}; & w_1 &= \frac{4}{6}; & w_2 &= \frac{1}{6} \\ \theta_1 &= \frac{1}{2}; & \theta_2 &= 1; & & \\ w_{10} &= \frac{1}{2}; & w_{20} &= -1; & w_{21} &= 2 \end{aligned} \quad ; \tau(h^3)$$

1.3.2.2. Método de Heun de Tercer Orden

$$\begin{aligned} w_0 &= \frac{1}{4}; & w_1 &= 0; & w_2 &= \frac{3}{4} \\ \theta_1 &= \frac{1}{3}; & \theta_2 &= \frac{2}{3}; & & \\ w_{10} &= \frac{1}{3}; & w_{20} &= 0; & w_{21} &= \frac{2}{3} \end{aligned} \quad ; \tau(h^3)$$

1.3.3. Métodos de Runge-Kutta de Cuarto Orden

$$\begin{aligned}
 y_{i+1} &= y_i + h\Phi(x_i, y_i) \\
 \Phi(x, y) &= w_0k_0 + w_1k_1 + w_2k_2 + w_3k_3 \\
 k_0 &= \varphi(x, y) \\
 k_1 &= \varphi(x + \theta_1h, y + (w_{10}k_0)h) \\
 k_2 &= \varphi(x + \theta_2h, y + (w_{20}k_0 + w_{21}k_1)h) \\
 k_3 &= \varphi(x + \theta_3h, y + (w_{30}k_0 + w_{31}k_1 + w_{32}k_2)h)
 \end{aligned}$$

1.3.3.1. Método de Kutta de Cuarto Orden

$$\begin{array}{cccc}
 w_0 = \frac{1}{6}; & w_1 = \frac{1}{3}; & w_2 = \frac{1}{3}; & w_3 = \frac{1}{6} \\
 \theta_1 = \frac{1}{2}; & \theta_2 = \frac{1}{2}; & \theta_3 = 1 & \\
 w_{10} = \frac{1}{2}; & w_{20} = 0; & w_{21} = \frac{1}{2} & \\
 w_{30} = 0; & w_{31} = 0; & w_{32} = 1 &
 \end{array}
 \quad ; \tau(h^4)$$

1.3.3.2. Método de cuarto orden asociado a la cuadratura de Newton-Cotes

$$\begin{array}{cccc}
 w_0 = \frac{1}{8}; & w_1 = \frac{3}{8}; & w_2 = \frac{3}{8}; & w_3 = \frac{1}{8} \\
 \theta_1 = \frac{1}{3}; & \theta_2 = \frac{2}{3}; & \theta_3 = 1 & \\
 w_{10} = \frac{1}{3}; & w_{20} = -\frac{1}{3}; & w_{21} = 1 & \\
 w_{30} = 1; & w_{31} = -1; & w_{32} = 1 &
 \end{array}
 \quad ; \tau(h^4)$$

1.3.3.3. Método de Gill

$$\begin{array}{cccc}
 w_0 = \frac{1}{6}; & w_1 = \frac{2}{6}(1 - \frac{1}{\sqrt{2}}); & w_2 = \frac{2}{6}(1 + \frac{1}{\sqrt{2}}); & w_3 = \frac{1}{6} \\
 \theta_1 = \frac{1}{2}; & \theta_2 = \frac{1}{2}; & \theta_3 = 1 & \\
 w_{10} = \frac{1}{2}; & w_{20} = (-\frac{1}{2} + \frac{1}{\sqrt{2}}); & w_{21} = (1 - \frac{1}{\sqrt{2}}) & \\
 w_{30} = 0; & w_{31} = -\frac{1}{\sqrt{2}}; & w_{32} = (1 + \frac{1}{\sqrt{2}}) &
 \end{array}
 \quad ; \tau(h^4)$$