Convergence Acceleration of Computer Methods for Grounding Analysis in Stratified Soils

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Abstract. The design of safe grounding systems in electrical installations is essential to assure the protection of the equipment, the power supply continuity and the security of the persons. In order to achieve these goals, it is necessary to compute the equivalent electrical resistance of the system and the potential distribution on the earth surface when a fault condition occurs. In the last years the authors have developed a numerical formulation based on the BEM for the analysis of grounding systems embedded in uniform and layered soils. As it is known, in practical cases the underlying series have a poor rate of convergence and the use of multilayer soils requires an out of range computational cost. In this paper we present an efficient technique based on the Aitken δ^2-process in order to improve the rate of convergence of the involved series expansions.

1. Introduction

From the beginnings of the large-scale use of electricity, one of the challenging stated problems has been to obtain the potential distribution of a grounding system. When a fault condition occurs, the grounding grid transports and dissipates the electrical currents produced into the ground, with the aim of ensuring that a person in the vicinity of the grounded installation is not exposed to a critical electrical shock, while preserving the continuity of the power supply and the integrity of the equipment. To achieve these goals, the equivalent electrical resistance of the system must be low enough to assure that fault currents dissipate mainly through the grounding grid into the earth. Moreover, electrical potential values between close points on the earth surface that can be connected by a person must be kept under certain maximum safe limits (step, touch and mesh voltages) [1, 2].

In the last four decades, several methods and procedures for the analysis and design of grounding grids have been proposed: methods based on the professional experience, on semi-empirical works, on experimental data obtained from scale model assays and laboratory tests, and even on intuitive ideas. Unquestionably, these contributions represented an important improvement in the grounding analysis area, although some problems have been systematically reported: the large computational costs required in the analysis of real cases, the unrealistic results obtained when segmentation of conductors is increased, and the uncertainty in the margin of error [1, 2, 3, 4], among others.

Maxwell's Electromagnetic Theory constitutes the starting point to obtain the mathematical equations that govern the dissipation of electrical currents into a soil. Nevertheless, although these equations are well-known for a long time, their application and resolution for the computing of grounding grids of large installations in practical cases present serious difficulties. First, it is obvious that no analytical solutions can be obtained for most of real problems. On the other hand, the characteristic geometry of grounding systems (a mesh of interconnected bare conductors with a relatively small ratio diameter/length) makes very difficult the use of numerical methods. Thus, the use of some widespread numerical techniques commonly applied for solving boundary value problems in engineering, such as finite elements or finite differences, is extremely expensive since the discretization of the domain (the ground excluding the electrode) is required. Consequently, obtaining sufficiently accurate results should imply unacceptable computing efforts in memory storage and CPU time.

In the last years, the authors have developed a numerical formulation based on the Boundary Element Method for the analysis of grounding systems. Thus, it is possible to derive specific computer methods of high accuracy for the analysis of grounding systems embedded in uniform soils models [5]. Besides, it is possible to explain rigorously the anomalous asymptotic behaviour of the clasical methods proposed for grounding analyis, and to identify the sources of error [4]. Its implementation in a Computer Aided Design application for grounding systems allows to analyze real grounding installations in real-time using conventional personal computers. Finally, this boundary element formulation has been extended for grounding grids embedded in layered soils [6, 7].

In 2005, the authors proposed a methodology for the analysis of a common and very important engineering problem in the grounding field: the problem of transferred earth potentials by grounding grids, that is, the existence of transferred earth potentials in a grounding installation for metallic stuctures or conductors connected or not connected to the grounding grid [1, 8]. The computer method based on the boundary element methodology for the case of uniform soil models can be found in [9], while the generalization to layered soil models can be found in [10].

In this paper we turn our attention to a problem which appears in the analysis of grounding grids in multilayer soils, related with the out-of-range computational requirements in some practical cases due to the poor rate of convergence of the series that appear when the method of images is applied to represent the different layers of soil. This topic should become the bottleneck of the whole computer method. In this work we propose the use of an efficient and mathematically well-founded extrapolation technique in order to accelerate the rate of convergence of the involved series expansions. This proposal will be presented in the framework of a general numerical approach based on the method of images to represent the layered soil model.

2. Mathematical model

The Maxwell's Electromagnetic Theory is the general framework to analyze the phenomena of the electrical current dissipation into the soil through a grounding grid. Thus, restricting the study to obtain the electrokinetic steady-state response and neglecting the inner resistivity of the conductors (so, potential is constant on the surface of the grounding electrode), the set of equations which governs the phenomena is given by

$$\mathrm{div}(\boldsymbol{\sigma}) = 0, \quad \boldsymbol{\sigma} = -\boldsymbol{\gamma}\,\mathbf{grad}(V) \text{ in } E; \;\; \boldsymbol{\sigma}^t \boldsymbol{n}_E = 0 \text{ in } \Gamma_E; \; V = V_\Gamma \text{ in } \Gamma; \; V \to 0, \text{ if } |\boldsymbol{x}| \to \infty \quad (1)$$

being E the earth, $\boldsymbol{\gamma}$ its conductivity tensor, Γ_E the earth surface, $\boldsymbol{n}_E$ its normal exterior unit field and Γ the surface of the grounded electrode [5]. Therefore, solutions of (1) are potential V and current density $\boldsymbol{\sigma}$ at an arbitrary point $\boldsymbol{x}$ when the electrode attains a voltage V_Γ (the Ground Potential Rise, or GPR) with respect to remote earth. The safety and design parameters of a grounding system can be then easily obtained from known values of V on Γ_E and $\boldsymbol{\sigma}$ on Γ [5, 7].

In many of the methods proposed for grounding analysis, the most common soil model considered is the homogeneous and isotropic one, where conductivity $\boldsymbol{\gamma}$ is substituted by an apparent scalar conductivity γ [1, 5]. Obviously, this model is valid if the soil is “essentially” uniform in all directions in the surroundings of the grounding grid. In order to take into account variations of the soil conductivity, particularly in depth, other models have been proposed [11]. Thus, the “layered soil models” consist in asuming the soil stratified in a number of layers, defined by an appropriate thickness and an apparent scalar conductivity that must be experimentally obtained. In fact, it is widely accepted that two-layer and three-layer soil models should be sufficient to obtain good and safe designs of grounding systems in most practical cases [1, 12, 13].

In the hypothesis of a stratified soil model formed by C layers with different conductivities, the problem (1) can be written in terms of the following Neumann exterior problem

$$\begin{gathered} \operatorname{div}(\boldsymbol{\sigma}_c) = 0, \quad \boldsymbol{\sigma}_c = -\gamma_c\, \mathbf{grad}(V_c) \text{ in } E_c,\ 1 \le c \le C; \\ \boldsymbol{\sigma}_1^t \boldsymbol{n}_E = 0 \text{ in } \Gamma_E;\ \ V_b = V_\Gamma \text{ in } \Gamma;\ V_c \to 0 \text{ if } |\boldsymbol{x}| \to \infty,\ 1 \le c \le C; \\ V_c = V_{c+1},\ 1 \le c \le C-1;\ \boldsymbol{\sigma}_c^t \boldsymbol{n}_c = \boldsymbol{\sigma}_{c+1}^t \boldsymbol{n}_c \text{ in } \Gamma_c,\ 1 \le c \le C-1; \end{gathered} \tag{2}$$

being b the layer in which the grounded electrode is buried, E_c each one of the soil layers, γ_c its scalar conductivity, V_c the potential at an arbitrary point in the layer E_c, $\boldsymbol{\sigma}_c$ its corresponding current density, Γ_c the interface between layers E_c and E_{c+1}, and $\boldsymbol{n}_c$ the normal field to Γ_c [7]. In this paper we restrict our analysis of the acceleration of convergence to two-layer soil models ($C = 2$), although it is straightforward to extend the analysis to other layered soil models.

On the other hand, if it is assumed that the earth surface Γ_E and the interfaces Γ_c between layers are horizontal (this hypothesis seems sound if we take into account the levelling and regularization processes performed in the surroundings of the substation site during the construction process of the electrical installation), then the application of the “method of images” and Green’s Identity [7] yields the following integral expression for potential $V_c(\boldsymbol{x}_c)$ at an arbitrary point $\boldsymbol{x}_c \in E_c$

$$V_c(\boldsymbol{x_c}) = \frac{1}{4\pi\gamma_b} \int\!\!\int_{\boldsymbol{\xi}\in\Gamma} k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi}) d\Gamma,\ \forall \boldsymbol{x}_c \in E_c, \tag{3}$$

where $\sigma(\boldsymbol{\xi})$ is the unknown leakage current density at any point $\boldsymbol{\xi}$ of the electrode surface $\Gamma \subset E_b$ ($\sigma = \boldsymbol{\sigma}^t \boldsymbol{n}$, where $\boldsymbol{n}$ is the normal exterior unit field to Γ), and $k_{bc}(\boldsymbol{x}_c, \boldsymbol{\xi})$ is the integral kernel formed by series of infinite terms corresponding to the resultant images obtained when Neumann exterior problem is transformed into a Dirichlet one [5, 7, 14]. Depending on the type of soil model considered (see the Appendix of reference [10], for example), these series can have a finite number of terms (e.g., for uniform soil models $C = 1$) or an infinite number of terms (e.g., the two-layer soil model $C = 2$).

At this point, it is important to remark that computing the potential distribution is only required on the earth surface Γ_E [1, 7]. So $c = 1$, it will be used in this paper from now on:

$$V_1(\boldsymbol{x_1}) = \frac{1}{4\pi\gamma_b} \int\!\!\int_{\boldsymbol{\xi}\in\Gamma} k_{b1}(\boldsymbol{x}_1, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi}) d\Gamma,\ \forall \boldsymbol{x}_1 \in \Gamma_E, \tag{4}$$

As it is known, weakly singular integral kernels depend on the inverse of the distances from the point $\boldsymbol{x}_1$ to the point $\boldsymbol{\xi}$ and to all its images with respect to the earth surface and to the interphases between layers; they also depend on the thickness and a ratio between conductivities of the layers [15, 16].

In the case of a two-layer model, this ratio is given by κ:

$$\kappa = \frac{\gamma_1 - \gamma_2}{\gamma_1 + \gamma_2} \tag{5}$$

and the singular kernels can be written in a general form

$$k_{b1}(\boldsymbol{x}_1, \boldsymbol{\xi}) = \sum_{n=0}^{\infty} k_{b1}^{[n]}(\boldsymbol{x}_1, \boldsymbol{\xi}), \quad k_{b1}^{[n]}(\boldsymbol{x}_1, \boldsymbol{\xi}) = \frac{\psi_n(\kappa)}{r(\boldsymbol{x}_1, \boldsymbol{\xi}_n)}, \tag{6}$$

where $\psi_n(\kappa)$ is a weighting coefficient that only depends on the ratio κ given by (5), and $r(\boldsymbol{x}_1, \boldsymbol{\xi}_n)$ is the Euclidean distance between the points $\boldsymbol{x}_1$ and $\boldsymbol{\xi}_n$, being $\boldsymbol{\xi}_0$ the point $\boldsymbol{\xi}$ on the electrode surface ($\boldsymbol{\xi}_0 = \boldsymbol{\xi}$), and being $\boldsymbol{\xi}_n$ ($n \neq 0$) the images of $\boldsymbol{\xi}$ with respect to the earth surface and to the interfaces between layers. The explicit expressions of these kernels can be found in the Appendix of the reference [10].

If one substitutes (6) in (4), it is clear that potential value at a point $\boldsymbol{x}_1 \in \Gamma_E$ can be computed by adding the contribution of each image

$$V_1(\boldsymbol{x_1}) = \sum_{n=0}^{\infty} V_1^{[n]}(\boldsymbol{x_1}) \tag{7}$$

where $V_1^{[n]}(\boldsymbol{x_1})$ is the potential contribution due to image n:

$$V_1^{[n]}(\boldsymbol{x_1}) = \frac{1}{4\pi\gamma_b} \int\int_{\boldsymbol{\xi}\in\Gamma} k_{b1}^{[n]}(\boldsymbol{x}_1, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi}) d\Gamma, \tag{8}$$

Different computer methods have been proposed in the literature for computing these potential contributions (8) [1, 2]. More specifically, in the last years the authors have proposed an efficient and well-founded numerical methodology based on the Boundary Element Method: with this approach it is possible to compute and design real grounding systems in the case of uniform and two-layer soil models [4, 5, 6, 7, 17], as well as analyze grounding related problems such as earth transferred potentials [9, 10].

In the next section we will deal with the problem of the rate of convergence of the infinite series involved in the integral kernels, and after analyzing the evolution of the error in the computation of potential, we will propose a method for accelerate the convergence of these series.

3. Convergence acceleration techniques of the series

In the case of layered soil models (and in particular for the 2-layer one), the series (6) involved in the calculus of potential (4) have a poor rate of convergence particularly when the ratio κ —given by (5)— is close to +1 or -1; i.e., when important differences between the electrical properties of the two layers of soil exist, which are usually the most interesting cases. It is important to remark that the increase in the computing cost by the use of multilayer soil models is justified when conductivities drastically vary, since two-layer models (or in general multilayer models) produce results noticeably different from those obtained by using a uniform soil model.

On the other hand, it is advisable to remind that in practice the computation of the potential distribution on the earth surface is usually the bottleneck of the complete process of grounding analysis, since it is necessary to compute the potential in an extremely high number of points on the earth surface in order to obtain high-quality results and to compute the safety parameters of the grounding grid: e.g., for a substation site of an approximated area of 40.000 m^2 it should be necessary to compute the value of potential in approximately 50.000 points by using formula (4). Consequently, it is very important to compute the potential accurately and low-costly from a computational point of view.

In this section, we present the techniques developed by the authors to increase the rate of convergence of the series: The starting point in the derivation of our proposal consists in studying the bound of the absolute error in the computing of potential values on the ground surface.

First of all, we will analyze a particular case which is common knowledge: the computing of potential for a punctual current source. This case will serve us to present a technique for accelerate the convergence of the infinite series involved in the integral kernels. Next we present the extension of the methodology to a general case of a grounding mesh.

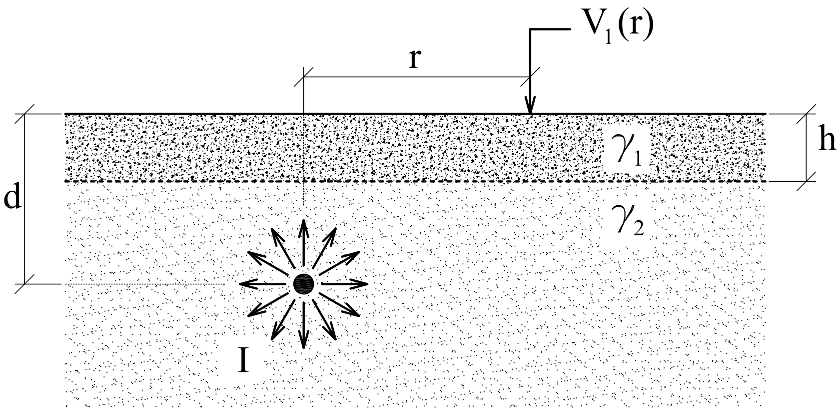

Figure 1. Scheme of a punctual source of current with intensity I buried to a depth d in a two-layer soil formed by an upper layer with a thickness h and conductivity γ_1, and a lower layer with conductivity γ_2.

Let consider a punctual source of current with intensity I buried to a depth d in a two-layer soil formed by an upper layer with a thickness h and conductivity γ_1, and a lower layer with conductivity γ_2 (Figure 1).

The potential V_1 on the ground surface is given by the following two expressions depending on the position of the source [15, 16, 18]: If it is placed in the upper layer then $d < h$, and potential is given by

$$V_1(r) = \frac{I}{2\pi d\gamma_1}\frac{1}{\sqrt{\widetilde{r}^2+1}} + \frac{I}{2\pi d\gamma_1}\sum_{n=1}^{\infty}\frac{\kappa^n}{\sqrt{\widetilde{r}^2+(2n\widetilde{h}-1)^2}} + \frac{I}{2\pi d\gamma_1}\sum_{n=1}^{\infty}\frac{\kappa^n}{\sqrt{\widetilde{r}^2+(2n\widetilde{h}+1)^2}} \quad (9)$$

being $\widetilde{r} = r/d$ and $\widetilde{h} = h/d$. If the punctual source is in the lower layer then $d > h$, and potential is given by

$$V_1(r) = \frac{I}{2\pi d\gamma_2}\sum_{n=0}^{\infty}\frac{(1-\kappa)\kappa^n}{\sqrt{\widetilde{r}^2+(2n\widetilde{h}+1)^2}} \quad (10)$$

In this paper, we have restricted our analysis to the case $d > h$, i.e. when the source is buried in the lower layer, but the study can be straightforwardly extended to the other case $d < h$.

Figure 2 shows the potential computed for two different types of 2-layer soil models: $\kappa = +0.998$ (corresponds to $\gamma_1 = 10^{-2}$ S/m and $\gamma_2 = 10^{-5}$ S/m) and $\kappa = -0.998$ (corresponds to $\gamma_1 = 10^{-5}$ S/m and $\gamma_2 = 10^{-2}$ S/m), for the geometric values: $\widetilde{h} = h/d = 0.25$ (e.g., corresponds to $d = 1$ m and $h = 0.25$ m) and $\widetilde{r} = 1$ (a point on the earth surface to a distance d over the vertical of the punctual source). The value of intensity I has been chosen $I = 2\pi d\gamma_2$ in order to represent directly the sum of the series in all graphics.

Now, let be ε_N the absolute error produced in the calculus of the potential by computing N terms of the series (i.e., by using the first N images): $\varepsilon_N = |V_1 - V_1^N|$, being V_1 the exact value and V_1^N the approximation by computing N terms of the series. In the case of potential (9), this absolute error is bounded by

$$\varepsilon_N < \left|\frac{I}{\pi d\gamma_1}\frac{(1-\kappa)\kappa^N}{\sqrt{\widetilde{r}^2+(2N\widetilde{h}-1)^2}}\right|; \text{if } \ d < h \quad (11)$$

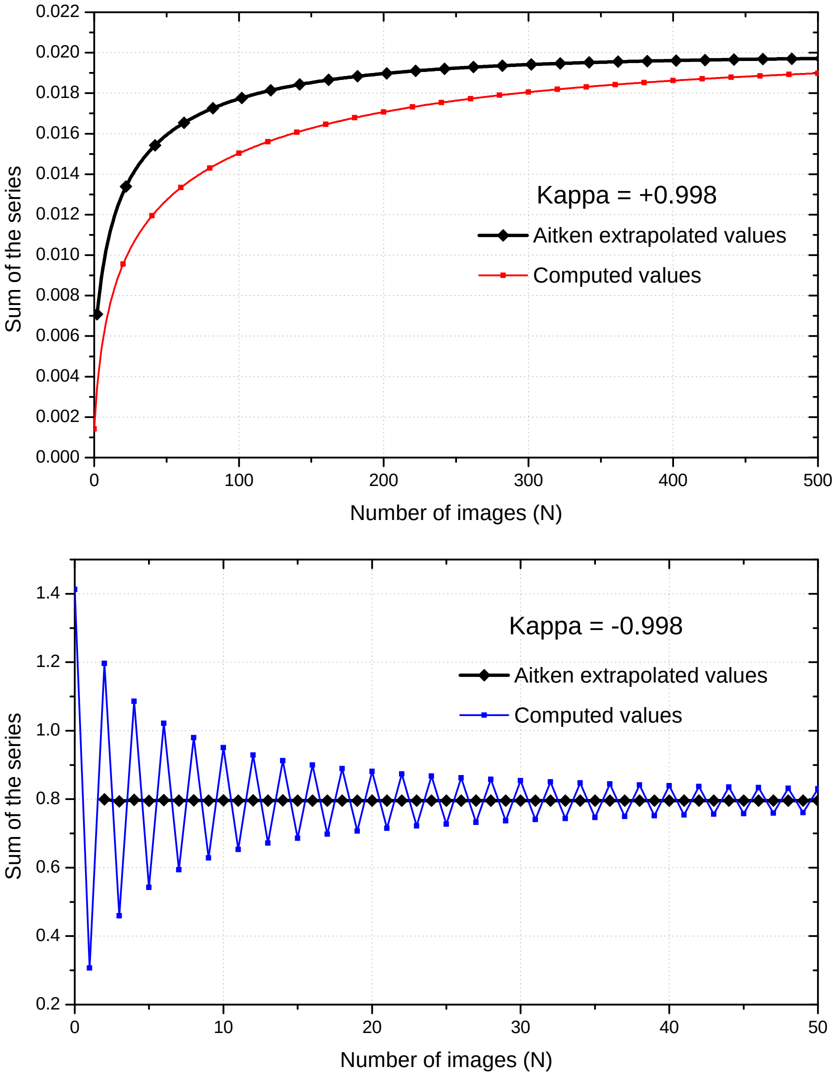

Figure 2. Potential on earth surface produced by a punctual current source buried to a depth d in a 2-layer soil of thickness of the upper layer h ($d > h$): Results depending on the number of images computed for a $\kappa = +0.998$ (up) and for a $\kappa = -0.998$ (down) for a ratio $\widetilde{r} = 1$ and $\widetilde{h} = 0.25$. It is also shown the values of the potential computed by using the proposed formula (14) based on the Aitken acceleration.

and consequently, for a large number of images N, the logarithm of ε_N is essentially linearly-dependent with N. In the case of potential (10), the error ε_N is bounded by

$$\varepsilon_N < \left| \frac{I}{2\pi d\gamma_2} \frac{\kappa^N}{\sqrt{\widetilde{r}^2 + (2N\widetilde{h} + 1)^2}} \right| ; \text{if} \quad d > h \tag{12}$$

and consequently, for a large number of images N, the logarithm of ε_N is again essentially linearly-dependent with N. Figure 3 shows the evolution of the absolute error in the computation of potential for the same previous two cases: $\kappa = +0.998$ and $\kappa = -0.998$. The linear dependency of the log-error is clear when the number of images N increases as predicted by formulae (11) and (12).

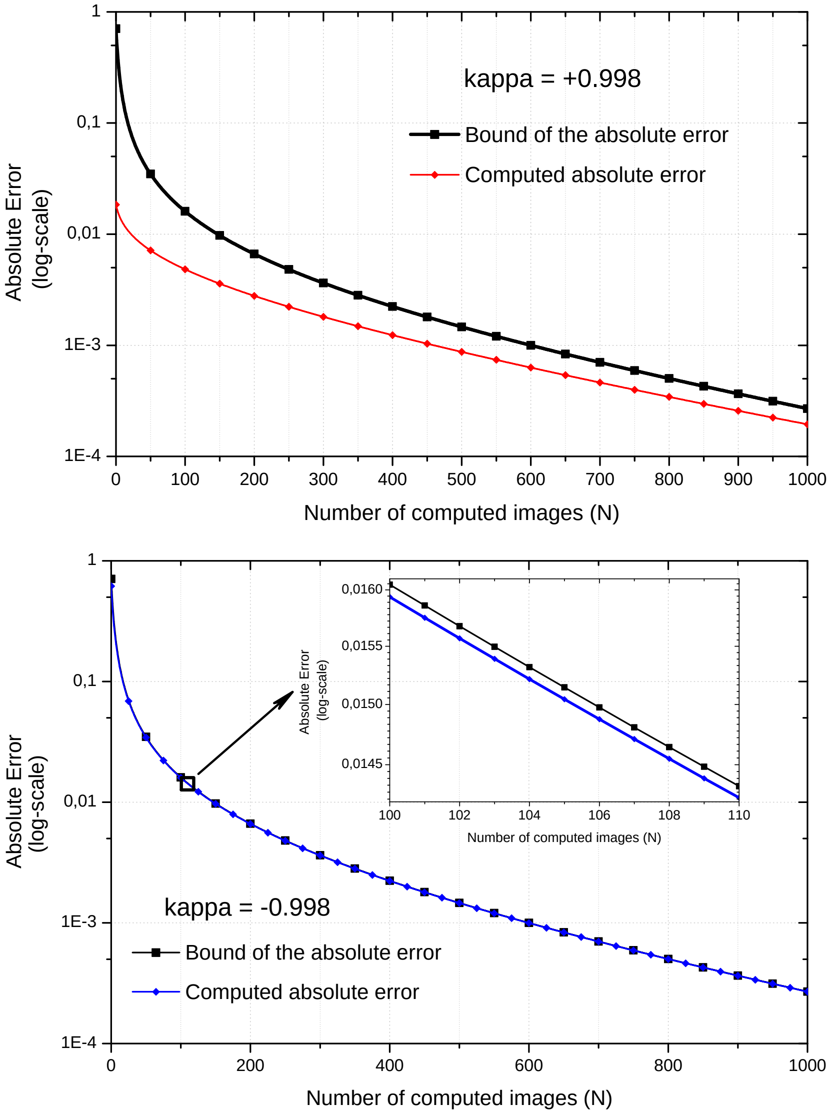

Figure 3. Absolute error ε_N in the potential computing on earth surface produced by a punctual current source buried to a depth d in a 2-layer soil of thickness of the upper layer h $(d > h)$: Results depending on the number of images computed for a $\kappa = +0.998$ (up) and for a $\kappa = -0.998$ (down) for a ratio $\tilde{r} = 1$ and $\widetilde{h} = 0.25$. It is also shown the bound of the absolute error predicted by expressions (11) and (12).

As we can observe from expressions (11) and (12), the bound of the absolute error (in logarithmic scale) is linear with N. Both are very important results. If the potential is computed by using two different numbers of terms of the series (namely N_1, N_2), the Richardson's deferred approach to the limit [19] allows to conclude that $\varepsilon_{N_2} = \varepsilon_{N_1} \kappa^{(N_2 - N_1)}$, that is, a geometric convergence is achieved since $|\kappa| < 1$. This expression should be used to obtain extrapolated values for the electrical potential, although it is possible to obtain better results by using the Aitken acceleration.

Due to this geometric convergence, the Aitken's δ^2-process [19] can be used to accelerate the convergence of the series and to obtain an *enhanced value of potential* (V^E). Thus, by using the

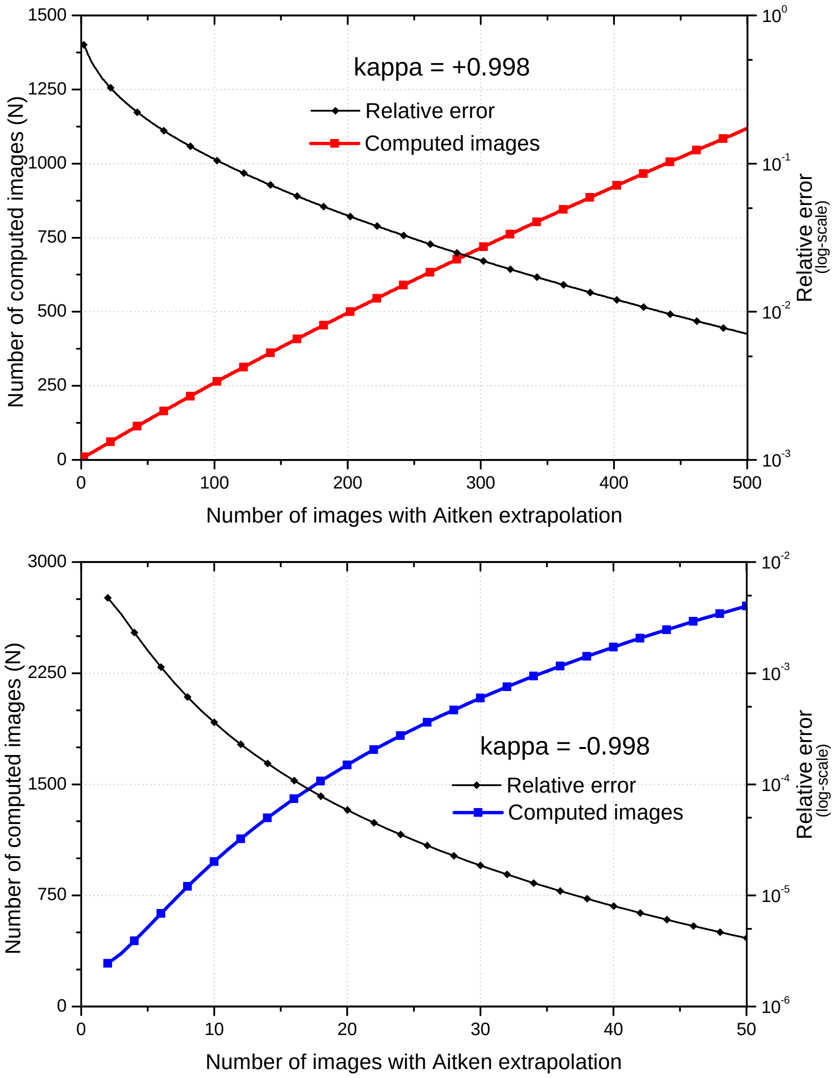

Figure 4. Number of images (left-scale) and relative error (right-scale) versus the number of images necessary for *potential improved* values by using the Aitken acceleration (14) on earth surface produced by a punctual current source buried to a depth d in a 2-layer soil of thickness of the upper layer h ($d > h$) computed for a $\kappa = +0.998$ (up) and $\kappa = -0.998$ (down) for a ratio $\widetilde{r} = 1$ and $\widetilde{h} = 0.25$.

computed values of the potential with three different numbers of terms of the series, namely N_1, N_2 and N_3 (satisfying $N_1 < N_2 < N_3$ and $N_3 - N_2 = N_2 - N_1$), the Aitken's process leads to

$$\log\left(\frac{\varepsilon_{N_3}}{\varepsilon_{N_2}}\right) = \log\left(\frac{\varepsilon_{N_2}}{\varepsilon_{N_1}}\right) \tag{13}$$

From this expression, it is straightforward to deduce a formula for computing an *enhanced* or *extrapolated* value of potential (V^E):

$$V^E = \frac{V^{N_1}V^{N_3} - V^{N_2}V^{N_2}}{V^{N_1} + V^{N_3} - 2V^{N_2}} \tag{14}$$

being V^{N_1}, V^{N_2} and V^{N_3} the computed values of the potential obtained by using N_1, N_2 and N_3 images.

This formula is very simple and easy to use: for a given point on the ground surface, three values of the potential (10) should be computed by using N_1, N_2 and N_3 number of terms of the series (satisfying $N_3 - N_2 = N_2 - N_1$) and then it is computed the *enhanced* value V^E by using the Aitken's δ^2 acceleration process given by (14).

As it can be seen, the figure 2 show the potential values and the extrapolated potential ones versus the number of images. It is important to remark the good quality of the enhanced values obtained by using the Aitken process computing with a few number of images, specially in the case of $\kappa = -0.998$. As it is obvious from expressions (9) and (10), values of $\kappa < 0$ always lead to alternate series which convergence can be amazingly accelerated by using formula (14): It is important to remind that negative values of κ use to be the most interesting cases in practice in the grounding substation design since they correspond to an upper layer less conductive than the lower layer ($\gamma_1 < \gamma_2$), e.g., gravel in the upper layer and clayey ground in the lower one.

Figures 4 show the number of images necessary to compute the potential if no extrapolation is used (number of computed images, N, in the left hand-side vertical axis) versus the number of images if Aitken-extrapolation is used (in the horizontal axis). It is also represented the relative error in the potential value (in the right hand-side vertical axis): e.g., in the case $\kappa = -0.998$, 10 images by using Aitken-formula are equivalent to compute $N = 1000$ images if no extrapolation is used and the relative error produced is less than $5 \cdot 10^{-4}$ (or 20 images using the Aitken process produce the same result in the evaluation of potential as the computing of $N = 1631$ images if no extrapolation is used, and the relative error is less than $6 \cdot 10^{-5}$).

In order to conclude this study, we have performed this analysis of acceleration of convergence by computing the potential in different points on the earth surface, and for different values of the thickness of the upper layer and conductivities of the layers. We have obtained the same improvement in the convergence of the series, and no significant difference has been observed: in fact, we have shown that the *ratio of acceleration* (i.e., the quotient between the number of terms required without using an acceleration process and the number of terms if Aitken process is applied) depends esentially on the tolerance on the relative error fixed as target, and not of the point where potential value is computed.

The techniques presented in the previous section to accelerate the convergence of the series can be applied to the different methods proposed in the bibliography for computing potential in the case of layered soil models based on the method of images. The authors have applied the Aitken δ^2-process to the Boundary Element numerical approach for grounding analysis derived in the last years for uniform and layered soil models [4, 5, 6, 7, 9, 10, 14]. In the analysis of real cases, the authors have obtained *speed-up factors* about 200 in the analysis of a total surface greater than 20000 m^2, computing the potential in almost 22000 points. Obviously the use of this acceleration of the convergence of the series allows to perform an accurate analysis of the grounding system in a layered soil model in real-time.

4. Conclusions

In this paper, the mathematical model of the physical phenomenon of the electrical current dissipation through a grounding grid into a stratified soil has been revisited. Furthermore, it has been presented a general methodology for the acceleration of the convergence of the series involved in the computing of potential in grounding analysis of layered soils, which is frequently the larger bottleneck in the computational cost of the computer earthing methods. The methodology for accelerating the convergence is based on the Aitken δ^2-process, being the starting point of its derivation the study of the potential produced by punctual current source.

This acceleration technique can be extended for computing the potential produced for a mesh of electrodes and vertical rods of a grounding grid. Its feasibility has been demonstrated by applying the proposed methodology to the analysis of a grounding system with a two-layer soil model. The improvement in the rate of convergence is spectacular, reducing two-orders of magnitude the CPU time in the analysis of a real earthing grid in a two layer soil model.

In our opinion, the use of acceleration techniques as the proposed in this paper opens the door to the use of layered models with more than two layers by application of the method of images for computing potential on the earth surface.

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