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High performance computing for analysis and postprocessing of earthing systems in layered soils

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Abstract

In this paper, we present a BEM formulation for the analysis and postprocessing of grounding systems embedded in layered soils, and we discuss the highlights of its implementation in a High-Performance Parallel Computer. The feasibility of this approach is demonstrated by its application to the analysis of a real grounding system with a two-layered soil model.

1 Introduction

From the early beginnings of the industrial use of electricity, obtaining the potential distribution when an electrical current is derived into the soil through a “grounded electrode” has been one of the challenging problems in the electrical engineering field. In most large electrical installations, the “grounding grid” consists of a mesh of interconnected cylindrical conductors, horizontally buried and supplemented by ground rods vertically thrust in certain places of the substation site. Its main objective is the transport and dissipation of electrical currents produced during fault conditions into the ground, ensuring that a person in the vicinity of the grounded installation is not exposed to a critical electrical shock, and preserving the continuity of the power supply and the integrity of the equipment. To attain these goals, the apparent electrical resistance of the grounding system must be low enough to guarantee that fault currents dissipate mainly through the grounding electrode into the soil. Moreover, the electrical potential differences between close points on the earth surface that can be connected by a person must be kept under certain maximum safe limits (step, touch and mesh voltages) [1].

In the last four decades, the operation of grounding systems has been extensively analyzed, and several methods for analysis and design have been proposed. Most of these methods are based on the professional experience, on semi-empirical works, on experimental data obtained from scale model assays and laboratory tests, or on intuitive ideas. Unquestionably, these contributions represented an important improvement in the grounding analysis area, although some problems have been systematically reported, such as the large computational costs required in the analysis of real cases, the unrealistic results obtained when segmentation of conductors is increased, and the uncertainty in the margin of error [1, 2, 3].

In the last years, the authors have developed a numerical formulation based on the BEM for the analysis of grounding systems with uniform soil models[4]. This approach allows to analyze real grounding installations in real-time using conventional computers. Recently, we have presented a generalization of the boundary element formulation for grounding grids embedded in layered soils [5]. This type of models is frequently used when there are important differences in the electrical properties of the soil. This is the case for example, when the excavation process during the construction of the substation produces a stratified soil, or as a consequence of a chemical treatment of the soil applied in the surroundings of the earthing system to improve the performance of the grounding electrode.

In this paper, we present a summary of this general BE approach. We also analyze the parallelization of the numerical formulation, and its application to a practical case by using the geometry of a real grounding grid.

2 Mathematical and Numerical Model

Maxwell's Electromagnetic Theory is the starting point to derive the equations that govern the dissipation of electrical currents into a soil. Thus, restricting the analysis to the electrokinetic steady-state response and neglecting the inner resistivity of the earthing conductors (then, potential can be assumed constant at every point of the grounding electrode surface), the 3D problem can be written as

$$\nabla \cdot \boldsymbol{\sigma} = 0, \boldsymbol{\sigma} = -\boldsymbol{\gamma} \nabla V \text{ in } E; \boldsymbol{\sigma}^t \mathbf{n}_E = 0 \text{ in } \Gamma_E; V = V_\Gamma \text{ in } \Gamma; V = 0 \text{ in } \Gamma_\infty \quad (1)$$

where E is the earth, $\boldsymbol{\gamma}$ is its conductivity tensor, Γ_E is the earth surface, \mathbf{n}_E is its normal exterior unit field and Γ is the electrode surface[4]. Therefore, the solution to (1) gives potential V and current density $\boldsymbol{\sigma}$ at an arbitrary point \mathbf{x} when the electrode attains a voltage V_Γ (Ground Potential Rise, or GPR) with respect to remote earth. Next, for known values of V on Γ_E and $\boldsymbol{\sigma}$ on Γ , it is straightforward to obtain the design and safety parameters of the grounding system [4].

The most commonly considered soil model in many of the proposed methods for grounding analysis is the homogeneous and isotropic one, where the conductivity tensor $\boldsymbol{\gamma}$ is substituted by an apparent scalar conductivity γ that must be experimentally obtained [1]. Obviously, this hypothesis can be used if the soil is "essentially uniform" in all directions in the vicinity of the grounding grid. Nevertheless, safety parameters involved in the grounding design can strongly vary if

the electrical properties of the soil change through out the substation site. Consequently, it is essential to develop more advanced models to take into account variations of the soil conductivity in the surroundings of the site.

Since considering all variations of the soil conductivity would be meaningless and unaffordable, some practical simplified soil models have been proposed. A family of these models (the “layered soil models”) consists in assuming that the soil is stratified in a number of horizontal layers, defined by an appropriate thickness and an apparent scalar conductivity that must be experimentally obtained [1]. Thus, if one considers that the soil is formed by L horizontal layers E_c ($c = 1, L$) with a different characteristic conductivity γ_c , while the grounded electrode is buried in the layer b , the mathematical problem (1) can be written in terms of a new exterior problem, which details can be found in [5].

As we have previously exposed, most grounding grids of real electrical substations consist of a mesh of interconnected bare cylindrical conductors, which ratio diameter/length-of-conductors uses to be relatively small ($\sim 10^{-3}$). This apparently simple geometry leads to serious troubles in the solution of the problem. Neither analytical solutions can be obtained, nor the most widespread numerical methods (such as FEM or FDM) can be used since the required discretization of the 3D domains E_c (excluding the grounding electrode) should involve a completely out of range computing effort. For these reasons, we turn our attention to other numerical techniques (BEM) which require only the discretization of the boundaries. So, it is necessary to derive an integral expression for the potential V in terms of unknowns defined on the boundary, that is, the electrode surface [4].

Thus, if one takes into account that the surroundings of the substations site are levelled and regularized during its construction (then the earth surface and the interfaces between layers can be assumed horizontal), the application of the “method of images” and Green’s Identity yields the following integral expression [5] for the potential $V_c(\mathbf{x}_c)$ at an arbitrary point $\mathbf{x}_c \in E_c$, in terms of the unknown leakage current density $\sigma(\boldsymbol{\xi})$ at any point $\boldsymbol{\xi}$ of the electrode surface $\Gamma \subset E_b$

$$V_c(\mathbf{x}_c) = \frac{1}{4\pi\gamma_b} \int \int_{\boldsymbol{\xi} \in \Gamma} k_{bc}(\mathbf{x}_c, \boldsymbol{\xi}) \sigma(\boldsymbol{\xi}) d\Gamma \quad \forall \mathbf{x}_c \in E_c, \quad (2)$$

where the integral kernels $k_{bc}(\mathbf{x}_c, \boldsymbol{\xi})$ are formed by infinite series of terms corresponding to the resultant images [5].

For a 2-layer case, for example, these kernels can be written in the general form

$$k_{bc}(\mathbf{x}_c, \boldsymbol{\xi}) = \sum_{l=0}^{\infty} k_{bc}^l(\mathbf{x}_c, \boldsymbol{\xi}), \quad k_{bc}^l(\mathbf{x}_c, \boldsymbol{\xi}) = \frac{\psi^l(\kappa)}{r(\mathbf{x}_c, \boldsymbol{\xi}^l(\boldsymbol{\xi}))}, \quad \kappa = \frac{\gamma_1 - \gamma_2}{\gamma_1 + \gamma_2} \quad (3)$$

where ψ^l is a weighting coefficient that only depends on the ratio κ defined in terms of the layer conductivities, and $r(\mathbf{x}_c, \boldsymbol{\xi}^l(\boldsymbol{\xi}))$ is the Euclidean distance between the points \mathbf{x}_c and $\boldsymbol{\xi}^l$. The points $\boldsymbol{\xi}^l$ ($l \neq 0$) are the images of $\boldsymbol{\xi}$ with respect to the earth surface and to the interfaces between layers and $\boldsymbol{\xi}^0(\boldsymbol{\xi}) = \boldsymbol{\xi}$ [5].

On the other hand, since expression (2) also holds on Γ , where the potential is given by the essential boundary condition ($V_b(\boldsymbol{\chi}) = V_\Gamma, \forall \boldsymbol{\chi} \in \Gamma$), the leakage

current density σ must satisfy a Fredholm integral equation of the first kind on Γ [4, 5], which variational form is given by the integral equation

$$\int \int_{\boldsymbol{\chi} \in \Gamma} w(\boldsymbol{\chi}) \left(\frac{1}{4\pi\gamma_b} \int \int_{\boldsymbol{\xi} \in \Gamma} k_{bb}(\boldsymbol{\chi}, \boldsymbol{\xi}) \sigma(\boldsymbol{\xi}) d\Gamma - V_\Gamma \right) d\Gamma = 0, \quad (4)$$

which must hold for all members $w(\cdot)$ of a class of functions defined on Γ [4]. It is important to remark that obtaining the leakage current density σ from (4) is the key of the problem, because next the potential at any point (and, of course, on the earth surface) can be straightforwardly obtained by means of (2).

The starting point in the development of the numerical model for solving the integral equation (4) is the discretization of the leakage current density σ and of the electrode surface Γ , for given sets of N trial functions $\{N_i(\boldsymbol{\xi})\}$ defined on Γ , and M boundary elements $\{\Gamma^\alpha\}$. Then, taking into account that kernels (3) are given by series, expression (2) for potential $V_c(\boldsymbol{x}_c)$ can also be discretized as

$$V_c(\boldsymbol{x}_c) = \sum_{i=1}^N \sigma_i V_{c,i}(\boldsymbol{x}_c), \quad V_{c,i}(\boldsymbol{x}_c) = \sum_{\alpha=1}^M \sum_{l=0}^{l_V} V_{c,i}^{\alpha l}(\boldsymbol{x}_c), \quad (5)$$

where l_V represents the number of terms that is necessary to consider until convergence is achieved, and $V_{c,i}^{\alpha l}(\boldsymbol{x}_c)$ depends on the integral on Γ^α of the kernel contribution $k_{bc}^l(\boldsymbol{x}_c, \boldsymbol{\xi})$ times the trial function $N_i(\boldsymbol{\xi})$ [5].

Furthermore, for a given set of N test functions $\{w_j(\boldsymbol{\chi})\}$ defined on Γ , the variational form (4) is reduced to the following linear system:

$$\sum_{i=1}^N R_{ji} \sigma_i = \nu_j \quad j = 1, \dots, N; \quad R_{ji} = \sum_{\beta=1}^M \sum_{\alpha=1}^M \sum_{l=0}^{l_R} R_{ji}^{\beta\alpha l}; \quad \nu_j = \sum_{\beta=1}^M \nu_j^\beta \quad (6)$$

where l_R represents the number of terms that is necessary to consider until convergence is achieved, $R_{ji}^{\beta\alpha l}$ depends on the integrals on Γ^α and on Γ^β of the kernel contribution $k_{bb}^l(\boldsymbol{\chi}, \boldsymbol{\xi})$ times the trial function $N_i(\boldsymbol{\xi})$ and times the test function $w_j(\boldsymbol{\chi})$, and ν_j^β depends on the integrals on Γ^β of the test function $w_j(\boldsymbol{\chi})$ [5].

As we can observe, the solution of system (6) provides the values of the unknowns σ_i ($i = 1, \dots, N$) that are necessary to compute the potential at any point on the earth surface by means of (5). Besides, the other safety parameters can be easily obtained from the potential distribution and the leakage current density σ [4].

In the present work, we focus our attention on the aspects related with the parallelization of the numerical algorithm for solving the linear system (6) and the postprocessing of the results, i.e., obtaining the potential distribution on the earth surface by using (5). In the references [4, 5], it can be found the whole development of the numerical formulation based on the BEM, including the derivation of a 1D approximated numerical approach (taking into account the real geometry of grounding systems in practical cases), and the highly efficient analytical integration techniques developed by the authors for computing terms $V_{c,i}^{\alpha l}(\boldsymbol{x}_c)$ of (5)

and $R_{ji}^{\beta\alpha l}$ of (6) which are finally computed by means of explicit formulae. Moreover, in [3, 4] a fully explicit discussion about the main numerical aspects of the BEM numerical approaches (such as the asymptotic convergence, the overall computational efficiency, and the complete explanation of the sources of error of the widespread intuitive methods) can be found.

Further discussion in this paper is restricted to the case of a Galerkin type weighting formulation, in which the matrix of coefficients in (6) is symmetric and positive definite [6], and we present an example corresponding to a two-layer soil model. Obviously, this BEM formulation can be applied to any other case with a higher number of layers. However, CPU time may increase exponentially, mainly because of the poor rate of convergence of the underlying series expansions.

3 Parallelization of the proposed BEM formulation

For a specific discretization (M elements of p nodes each, and a total number of N degrees of freedom), the computational effort of the proposed BEM formulation should be evaluated by analyzing the two main phases of the computing: the generation of linear system (6) and its solution, and the postprocessing, i.e., obtaining the potential distribution on the earth surface by using (5).

In reference with the matrix generation, this process requires $O(N^2 p^2 / 2)$ operations (since p^2 series of contributions $R_{ji}^{\beta\alpha l}$ have to be computed for every pair of elements and approximately half of them are discarded because of symmetry). While in uniform soil models these series are formed by only two terms, in 2-layer models the series have an infinite number of terms (as shown in (6)), that will be numerically added up until an upper limit of summands (l_R) is achieved or a tolerance is fulfilled.

The linear system solving process requires $O(N^3 / 3)$ operations (since the matrix is symmetric and full) if the resolution is carried out with a direct method. This would be unaffordable in large problems. Then, a semiiterative technique should be preferred. In our case, a diagonal preconditioned conjugate gradient algorithm with assembly of the global matrix [4] has been proved to be extremely efficient for solving large scale problems with a very low computational cost in comparison with matrix generation. So, the cost of the system resolution should never prevail.

On the other hand, once the leakage current density has been obtained by solving (6), the cost of computing the potential at any given point by means of (5) only requires $O(Mp)$ operations, since p series of contributions $V_{c,i}^{\alpha l}(\mathbf{x}_c)$ have to be computed for every element. Now, as in the matrix generation process, a 2-layer soil model requires the evaluation of infinite series (see (5)), that will be numerically added up until an upper limit of summands (l_V) is achieved or a tolerance is fulfilled. Furthermore, if it is necessary to compute potentials at a large number of points (i.e. to draw potential contours on the earth surface), the computing time may shoot-up and become very important.

So, the most critical time-consuming process of this BEM numerical formulation is the matrix generation, followed by the computation of the potential at a large

number of points (once the leakage current density has been obtained). Both processes accept massive parallelization. Consequently, CPU time could be reduced under acceptable levels even for cases of extremely large models, if the number of available processors is high enough, in spite of the efficiency losses due to the data transfer overhead and the system administration workload. Next we will discuss the parallelization (i.e., the distribution of different tasks of the program among several processors) of the matrix generation process in a 2-layer case. Obviously, the discussion and conclusions for the postprocessing task are very similar—in fact, its parallelization is easier than in the matrix generation process, since there is only one loop over the whole M elements in (5)—and it will not be presented now.

The BEM numerical approach has been implemented on a CAD system, which has been compiled and run onto an *Origin 2000 Silicon Graphics* computer at the *European Center for Parallelism of Barcelona* (CEPBA). The compilation process of the code has been made in sequential and parallel modes, and the executions have been run for the uniform and the two-layer models. The O2000 used in this work is a High-Performance Computer with 64 MIPS R10000 processors at 250 MHz. It has a peak performance of 32 GFlops. Internally, the O2000 is organized in clusters of 2 processors sharing a main memory of 256 Mbyte. Each processor has 4 Mbyte of cache memory. The clusters are connected by an hypercube network. Each processor can access all the distributed main memory through the network. Then, the O2000 can be programmed as an 8 Gbyte shared memory machine. The input/output devices have a capability of 1.2 Gbytes/s.

In our analysis, we have used compiler directives in the parallelization mode, according to the present OpenMP standard. Several reasons justify this selection: shared memory computer necessary for using this mode is available; the use of compiler directives grants clearness to a parallel code that may be handled in the future; the OpenMP syntax ensures the portability of the parallel code; and, as we will present next, the loop to be run in parallel is transformable into an adequate form so that directives are more efficient.

Going back to the analysis of the matrix generation process, equation (6) shows that it is performed by means of a double loop that couples every element with all the other ones ($M(M + 1)/2$ cycles). Into each cycle, the elemental matrix corresponding to a pair of elements is calculated and immediately assembled into the system matrix. If we try to parallelize this double loop, we find that the assembly of the elemental matrices causes a dependency between the actions of the threads or processes. This drawback can be avoided by taking the assembly process out of that loop which implies, first, the computation and the storage of all the elemental matrices and, second, the assembly in a sequential mode. This scheme requires approximately twice more memory space than the original one, but in any case this memory space is not very large.

Hence our objective is the nested DO loops for computing the elemental matrices. There are two options: the parallelization of the outer loop or the inner one. Next we will analyze both: Figure 1 shows the evolution of the speed-up factor obtained with different number of processors for both types of parallelization in a

2-layer grounding analysis (the speed-up factor has been referenced to the sequential CPU time). These results were obtained with the schedule option “Dynamic,1” and the strict value was approximated by taking the minimum of 4 CPU time measures made for the same option. The variance of the four measures was very small, anyway, in all cases.

When the outer loop is parallelized, results are much better because the granularity is bigger in that way, and so, the cost of managing the parallel execution is lower: since the numerical approach leads to a symmetric formulation, the coupling of every element of the grid with each one of the others can be represented by a triangle of M columns, of which the first one has M rows and the last one has 1 row. Therefore in the case of the parallelization of the outer loop, the columns of the triangle (i.e., the cycles of the outer loop) are distributed among the processors, while in the case of the parallelization of the inner loop, the rows of one column are distributed among the processors. In this case, when computations on that column are finished the program moves sequentially to the next one, where another distribution of its rows among the processors is performed. This effect of granularity is, of course, more sensible when the number of processors grows, as it can be seen in figure 1.

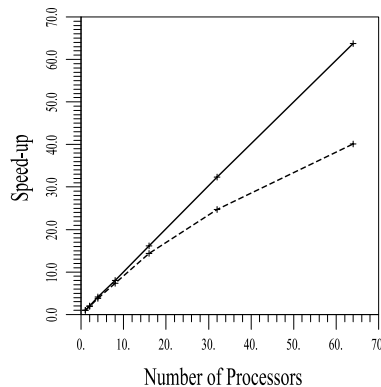


Figure 1: Parallelization of the outer (continuous) and inner loops (discontinuous).

As we have shown, the parallelization of the outer loop is more profitable than the parallelization of the inner one. The cycles in this case will be the columns of that triangle that couples one element to the other ones. The way to distribute these cycles (that have very different sizes in practice) among the processors becomes a critical decision.

Finally, we study the speed-up factors (referenced to the sequential CPU time) obtained for the outer-loop parallelization with different number of processors by using different “schedule” in the OpenMP compiler directives options. “Static” schedules with a high chunk (i.e., the number of cycles in a task) are the less profitable ones, since the size of the cycles decreases linearly. When no chunk value is

specified, all the columns are uniformly distributed in the beginning. “Dynamic” schedules improve this behaviour because when one processor finishes a task, it dynamically takes the next one. Best results are obtained for a “dynamic” schedule with a chunk parameter of 1 column. This is the most lively scheme, since there are never waiting processors, although it requires the biggest amount of parallelization management. “Guided” schedules distribute initially all the columns among all the processors into pieces with exponentially varying size. In this case, results are very similar to those obtained with the “dynamic” ones. In general, for any schedule, we obtained worse results when the chunk parameter and the number of processors are high, because then, some processors do not get any work. Summarizing, speed-up factors obtained for the outer parallelization are very close to the number of processors for good schedules, that is, “dynamic” or “guided” with low chunk parameters, so we can ensure that the parallelization of this loop is very profitable and the best option.

4 Application to a Practical Case

Next we present the analysis of a substation grounding considering 3 different soil models by using the presented parallelized BEM approach. The earthing grid is formed by a mesh of 107 cylindrical conductors (diameter: 11.28 mm) buried to a depth of 80 cm, supplemented with 67 vertical rods (each one has a length of 1.5 m and a diameter of 14.0 mm) [4]. The Ground Potential Rise considered has been 10 kV, and the numerical model used in the analysis has been a Galerkin formulation with a discretization in 241 elements.

The different soil models considered are the following: Model A is a uniform soil model with a resistivity of $50\Omega\text{ m}$; Model B is a 2-layered soil model formed by an upper layer of $400\Omega\text{ m}$ and a thickness of 70 cm and a lower layer with a resistivity of $50\Omega\text{ m}$ (consequently, all electrodes of the grounding grid are buried in the lower layer); and Model C is also a 2-layered soil model formed by an upper layer of $400\Omega\text{ m}$ and a thickness of 1 m and a lower layer with a resistivity of $50\Omega\text{ m}$ (consequently, most electrodes of the grounding grid are buried in the upper layer while part of the vertical rods are in the lower layer).

Figure 2 shows the potential distribution ($\times 10\text{ kV}$) on the earth surface obtained by using the soil models A, B and C. Furthermore it is possible to obtain the Equivalent Resistance of the grounding system and the Total Current that flows to the ground for each case: $0.336597\ \Omega$ and 29.7091 kA for the Model A; $0.352218\ \Omega$ and 28.3915 kA for the Model B; and $0.485985\ \Omega$ and 20.5768 kA for the Model C. As it is shown, results noticeably vary when different soil models are used, and in consequence, the grounding design parameters (Equivalent Resistance, Touch-Voltage, Step-Voltage and Mesh-Voltage) significantly change. For this reason, in those cases where conductivity changes markedly with depth, it is essential to perform the analysis of the grounding system with this BEM technique (although the computing cost increases) in order to ensure the safety of the installation.

Table 1 shows the CPU times and speed-ups obtained in the matrix generation process of the grounding analysis for the 3 soil models by using 1, 2, 4 and 8

processors. In some cases, we have obtained speed-ups bigger than the number of processors, due to small errors in the measurement of CPU time by the processors, and to the additional optimization of the code that the parallel compiler introduces. Finally, it is important to notice that the increase in the CPU time in the model C is due to the type of soil model considered. In this case, a part of vertical rods are in the upper layer and other part in the lower, and consequently integral kernels combining electrodes in different layers have to be used [5]. These integral kernels are different from those used in the model B (all the electrodes are in the same layer) and they involve series with a lower rate of convergence.

Table 1: CPU Time (in s) and Speed-up by using 1,2,4 and 8 processors

Model	1 Proc.	S-p	2 Proc.	S-p	4 Proc.	S-p	8 Proc.	S-p
A	2.44	1	—	—	—	—	—	—
B	81.26	1	40.85	1.98	20.41	3.98	10.09	8.05
C	443.28	1	218.10	2.03	111.38	3.98	53.53	8.28

5 Conclusions

The authors have developed a BEM approach for the analysis of grounding systems embedded in stratified soils that accepts massive parallelization. The proposed formulation has been implemented in a high-performance parallel computer (HPPC), and the code has been applied to the analysis of a real grounding system. The results prove that the proposed multi-layer BEM formulation will become a real-time design tool in a close future, as high-performance parallel computing becomes a widespread available resource in engineering.

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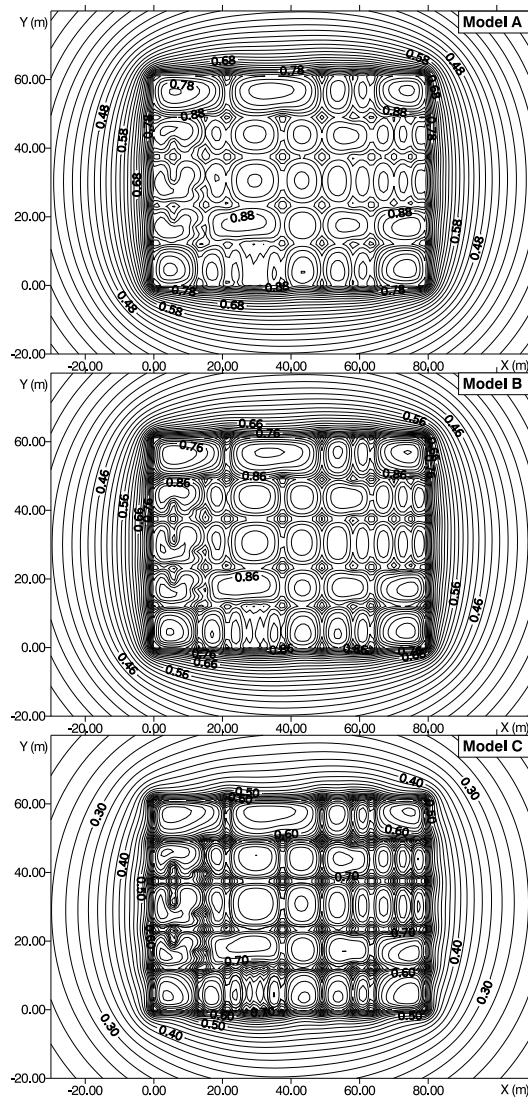


Figure 2: Potential distribution ($\times 10\text{kV}$) on the earth surface for different soil models