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A general formulation based on the boundary element method for the analysis of grounded instalations in layered soils

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Abstract

The design of safe grounding systems requires computing the potential level distribution on the earth surface for reasons of human security, as well as the equivalent resistance of the earthing installation for reasons of equipment protection (Sverak *et al.*[1], ANSI/IEEE[2]).

In the last three decades several methods for grounding analysis have been proposed, most of them based on practice and intuitive ideas. Although these techniques represented an important improvement in this area, some problems such as large computational requirements, unrealistic results when segmentation of conductors is increased, and uncertainty in the margin of error, were reported (Sverak *et al.*[1], ANSI/IEEE[2], Garret & Pruitt[3]).

Navarrina *et al.*[4] and Colominas *et al.*[5] have developed in the last years a general boundary element formulation for grounding analysis in uniform soils, in which these intuitive methods can be indentified as particular cases. Furthermore, starting from this BE numerical approach, more efficient and accurate formulations have been developed and succesfully applied (with a very reasonable computational cost) to the analysis of large grounding systems in electrical substations.

In this paper we present a new Boundary Element formulation for substation grounding systems embedded in layered soils. The feasibility of this BEM approach for two-layer soil models is demonstrated by solving a real application problem.

1 Introduction

Physical phenomena of fault currents dissipation into the earth can be described by means of Maxwell's Electromagnetic Theory (Durand[6]). If one constrains the analysis to the obtention of the electrokinetic steady-state response and neglects the inner resistivity of the earthing conductors —therefore, potential is assumed constant in every point of the grounding electrode surface—, the 3D problem can be written as

$$\begin{gathered}\mathbf{div}\boldsymbol{\sigma} = 0, \qquad \boldsymbol{\sigma} = -\boldsymbol{\gamma}\,\mathbf{grad}V \quad \text{in} \quad E; \\ \boldsymbol{\sigma}^t \boldsymbol{n}_E = 0 \quad \text{in} \quad \Gamma_E; \quad V = V_\Gamma \quad \text{in} \quad \Gamma; \quad V \longrightarrow 0, \quad \text{if} \quad |\boldsymbol{x}| \to \infty;\end{gathered} \tag{1}$$

where E is the earth, $\boldsymbol{\gamma}$ its conductivity tensor, Γ_E the earth surface, $\boldsymbol{n}_E$ its normal exterior unit field and Γ the electrode surface (Navarrina *et al.*[4], Colominas[7]). The solution to this problem gives potential V and current density $\boldsymbol{\sigma}$ at an arbitrary point $\boldsymbol{x}$ when the electrode attains a voltage V_Γ (Ground Potential Rise, or GPR) relative to a distant grounding point.

Most of the methods proposed are founded on the hypothesis that soil can be considered homogeneous and isotropic. Therefore, $\boldsymbol{\gamma}$ is substituted by an apparent scalar conductivity γ, that can be experimentally obtained (Sverak *et al.*[1]). In general, if the soil is essentially uniform (horizontally and vertically) in the surrondings of the grounding grid, this assumption does not introduce significant errors (ANSI/IEEE[2]). Nevertheless, since grounding design parameters can significantly change as soil conductivity varies, it is necessary to develop more accurate models that take into account the variation of soil conductivity in the surroundings of the substation site.

Obviously, from a technical (and also economical) point of view, the development of models to describe all variations of soil conductivity in the surroundings of a grounding system would be unaffordable. For this reason, a more practical and quite realistic approach to situations where conductivity is not markedly uniform with depth is to consider the earth stratified in a number of horizontal layers, each one with an appropriate thickness and apparent scalar conductivity. In fact, in most cases, an equivalent two-layer soil model is sufficient to obtain safe designs of grounding systems (ANSI/IEEE[2]).

When the grounding electrode is buried in the upper layer, (1) can be reduced to the Neumann Exterior Problem:

$$\begin{gathered}\Delta V_1 = 0 \quad \text{in} \quad E_1, \quad \Delta V_2 = 0 \quad \text{in} \quad E_2, \quad V_1 = V_\Gamma \quad \text{in} \quad \Gamma, \\ \frac{dV_1}{dn} = 0 \quad \text{in} \quad \Gamma_E, \quad \gamma_1 \frac{dV_1}{dn} = \gamma_2 \frac{dV_2}{dn} \quad \text{in} \quad \Gamma_L, \\ V_1 = V_2 \quad \text{in} \quad \Gamma_L, \quad V_1 \longrightarrow 0 \quad \text{and} \quad V_2 \longrightarrow 0 \quad \text{if} \quad |\boldsymbol{x}| \to \infty,\end{gathered} \tag{2}$$

being E_1 and E_2 the upper and lower layers of the earth, Γ_L the interface between them, γ_1 and γ_2 the apparent scalar conductivities of both layers, and V_1 and V_2 the potential in each layer (Aneiros[8]). In this paper, we restrict the further development to the above case. The analogous statement to (2) when the grounding grid is buried in the lower soil layer can be found in Aneiros[8] and Colominas *et al.*[9].

2 Variational Statement of the Problem

Grounding systems in most of real electrical substations consist of a grid of interconnected bare cylindrical conductors, horizontally buried and supplemented by a number of vertical rods, which ratio diameter/lenght uses to be relatively small ($\sim 10^{-3}$). This particular geometry precludes to obtain analytical solutions, and the use of standard numerical techniques (such as Finite Differences or Finite Elements) that requires the discretization of domains E_1 and E_2, implies an out of range computational effort. At this point, since computation of potential is only required on Γ_E, and the equivalent resistance can be easily obtained in terms of the leakage current density $\sigma = \boldsymbol{\sigma}^t \boldsymbol{n}$ on Γ, being $\boldsymbol{n}$ the normal exterior unit field to Γ, we turn our attention to a boundary element approach, which will only require the discretization of grounding surface Γ (Colominas[7]).

On the other hand, if one takes into account that surroundings of substation site are levelled during its construction, earth surface Γ_E and the interface between the two soil layers Γ_L can be assumed horizontal (Aneiros[8]). Thus, application of method of images (symmetry) and Green's Identity to problem (2) yield the following expressions for potential $V_1(\boldsymbol{x}_1)$ and $V_2(\boldsymbol{x}_2)$, at arbitrary points $\boldsymbol{x}_1$ in E_1 and $\boldsymbol{x}_2$ in E_2, in terms of the unknown leakage current density $\sigma(\boldsymbol{\xi})$ at any point $\boldsymbol{\xi} \equiv [\xi_x, \xi_y, \xi_z]$ on Γ:

$$V_1(\boldsymbol{x}_1) = \frac{1}{4\pi\gamma_1} \iint_{\boldsymbol{\xi}\in\Gamma} k_{11}(\boldsymbol{x}_1, \boldsymbol{\xi})\, \sigma(\boldsymbol{\xi}) d\Gamma, \ \forall \boldsymbol{x}_1 \in E_1, \tag{3}$$

being $k_{11}(\boldsymbol{x}_1, \boldsymbol{\xi})$ the weakly singular kernel

$$\begin{aligned} k_{11}(\boldsymbol{x}_1, \boldsymbol{\xi}) = & \frac{1}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, \xi_z])} + \frac{1}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, -\xi_z])} \\ & + \sum_{i=1}^{\infty} \Big[\frac{\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, 2iH + \xi_z])} + \frac{\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, 2iH - \xi_z])} \\ & + \frac{\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, -2iH + \xi_z])} + \frac{\kappa^i}{r(\boldsymbol{x}_1, [\xi_x, \xi_y, -2iH - \xi_z])} \Big]; \end{aligned} \tag{4}$$

and

$$V_2(\boldsymbol{x}_2) = \frac{1}{4\pi\gamma_1}\iint_{\boldsymbol{\xi}\in\Gamma} k_{12}(\boldsymbol{x}_2,\boldsymbol{\xi})\,\sigma(\boldsymbol{\xi})d\Gamma,\ \forall\boldsymbol{x}_2\in E_2, \tag{5}$$

with weakly singular kernel $k_{12}(\boldsymbol{x}_2,\boldsymbol{\xi})$

$$\begin{aligned} k_{12}(\boldsymbol{x}_2,\boldsymbol{\xi}) = &\frac{1+\kappa}{r(\boldsymbol{x}_2,[\xi_x,\xi_y,\xi_z])} + \frac{1+\kappa}{r(\boldsymbol{x}_2,[\xi_x,\xi_y,-\xi_z])} \\ &+ \sum_{i=1}^{\infty}\Big[\frac{(1+\kappa)\kappa^i}{r(\boldsymbol{x}_2,[\xi_x,\xi_y,2iH+\xi_z])} + \frac{(1+\kappa)\kappa^i}{r(\boldsymbol{x}_2,[\xi_x,\xi_y,2iH-\xi_z])}\Big]; \end{aligned} \tag{6}$$

where $r(\boldsymbol{x},[\xi_x,\xi_y,\xi_z])$ indicates the distance from $\boldsymbol{x}$ to $\boldsymbol{\xi}\equiv[\xi_x,\xi_y,\xi_z]$ —and to symmetric points of $\boldsymbol{\xi}$ with respect to Γ_E and Γ_L, which distances appear in different terms in kernels (4) and (6)—, H is the heigth of the upper soil layer, and κ is a relation between conductivities of both layers: $\kappa = (\gamma_1-\gamma_2)/(\gamma_1+\gamma_2)$ —Colominas *et al.*[9]—.

Now, the application of boundary condition $V_1(\boldsymbol{\chi}) = 1$, $\forall\boldsymbol{\chi}\in\Gamma$ (since V and $\boldsymbol{\sigma}$ are proportional to the GPR value, we can use the normalized boundary condition $V_\Gamma = 1$) leads to a Fredholm integral equation of the first kind defined on Γ, in terms of the leakage current density σ (Colominas[5]). A weaker variational form of this equation can be written as:

$$\iint_{\boldsymbol{\chi}\in\Gamma} w(\boldsymbol{\chi})\left(\frac{1}{4\pi\gamma_1}\iint_{\boldsymbol{\xi}\in\Gamma} k_{11}(\boldsymbol{\chi},\boldsymbol{\xi})\,\sigma(\boldsymbol{\xi})\,d\Gamma - 1\right)d\Gamma = 0, \tag{7}$$

which must hold for all members $w(\boldsymbol{\chi})$ of a suitable class of test fuctions defined on Γ. Obviously, a Boundary Element approach seems to be the right choice to solve equation (7).

3 Boundary Element Formulation

For a given set of $\mathcal{N}$ trial functions $\{N_i(\boldsymbol{\xi})\}$ defined on Γ, and for a given set of $\mathcal{M}$ 2D boundary elements $\{\Gamma^\alpha\}$, the unknown leakage current density σ and the grounding electrode surface Γ can be discretized in the form,

$$\sigma(\boldsymbol{\xi}) = \sum_{i=1}^{\mathcal{N}}\sigma_i\,N_i(\boldsymbol{\xi}), \qquad \Gamma = \bigcup_{\alpha=1}^{\mathcal{M}}\Gamma^\alpha, \tag{8}$$

and expressions (3) and (5) can be approximated as

$$V_1(\boldsymbol{x}_1) = \sum_{i=1}^{\mathcal{N}}\sigma_i\,V_{1i}(\boldsymbol{x}_1), \qquad V_{1i}(\boldsymbol{x}_1) = \sum_{\alpha=1}^{\mathcal{M}}V_{1i}^\alpha(\boldsymbol{x}_1), \quad \forall\boldsymbol{x}_1\in E_1; \tag{9}$$

$$V_2(\boldsymbol{x}_2) = \sum_{i=1}^{\mathcal{N}} \sigma_i \, V_{2i}(\boldsymbol{x}_2), \qquad V_{2i}(\boldsymbol{x}_2) = \sum_{\alpha=1}^{\mathcal{M}} V_{2i}^{\alpha}(\boldsymbol{x}_2), \quad \forall \boldsymbol{x}_2 \in E_2; \tag{10}$$

being potential coefficients V_{1i}^{α} and V_{2i}^{α},

$$V_{1i}^{\alpha}(\boldsymbol{x}_1) = \frac{1}{4\pi\gamma_1} \int\int_{\boldsymbol{\xi}\in\Gamma^{\alpha}} k_{11}(\boldsymbol{x}_1, \boldsymbol{\xi}) \, N_i(\boldsymbol{\xi}) \, d\Gamma^{\alpha}, \quad \forall \boldsymbol{x}_1 \in E_1; \tag{11}$$

$$V_{2i}^{\alpha}(\boldsymbol{x}_2) = \frac{1}{4\pi\gamma_1} \int\int_{\boldsymbol{\xi}\in\Gamma^{\alpha}} k_{12}(\boldsymbol{x}_2, \boldsymbol{\xi}) \, N_i(\boldsymbol{\xi}) \, d\Gamma^{\alpha}, \quad \forall \boldsymbol{x}_2 \in E_2. \tag{12}$$

Moreover, for a given set of $\mathcal{N}$ test functions $\{w_j(\boldsymbol{\chi})\}$ defined on Γ, the variational statement (7) is reduced to the system of linear equations

$$\sum_{i=1}^{\mathcal{N}} R_{ji}\sigma_i = \nu_j, \qquad j = 1, \ldots, \mathcal{N}; \tag{13}$$

$$R_{ji} = \sum_{\beta=1}^{\mathcal{M}} \sum_{\alpha=1}^{\mathcal{M}} R_{ji}^{\beta\alpha}, \qquad \nu_j = \sum_{\beta=1}^{\mathcal{M}} \nu_j^{\beta}; \tag{14}$$

$$R_{ji}^{\beta\alpha} = \frac{1}{4\pi\gamma_1} \int\int_{\boldsymbol{\chi}\in\Gamma^{\beta}} w_j(\boldsymbol{\chi}) \int\int_{\boldsymbol{\xi}\in\Gamma^{\alpha}} k_{11}(\boldsymbol{\chi}, \boldsymbol{\xi}) \, N_i(\boldsymbol{\xi}) \, d\Gamma^{\alpha} d\Gamma^{\beta} \tag{15}$$

$$\nu_j^{\beta} = \int\int_{\boldsymbol{\chi}\in\Gamma^{\beta}} w_j(\boldsymbol{\chi}) \, d\Gamma^{\beta}. \tag{16}$$

It is important to notice that expression (15) is satisfied when electrodes α and β are buried in the upper layer. Obviously, if a part of the earthing grid is in the lower layer ($\boldsymbol{\chi} \in E_2$), the integral kernel must be substituted by $k_{12}(\boldsymbol{\chi}, \boldsymbol{\xi})$ in (6) —Aneiros[8]—.

In real problems, the discretization required to solve the above equations would imply a large number of degrees of freedom. On the other hand, the coefficient matrix in (13) is full and each contribution (15) requires an extremely high number of evaluations of the kernel and double integration on a 2D domain. For these reasons, some additional simplifications in the BEM approach must be introduced to reduce the computational cost.

4 Approximated 1D BE Formulation

Taking into account the kind, size and disposition of the electrodes of a grounding system in most of electrical substations, it is possible to assume that the leakage current density is constant around the cross section of the cylindrical electrode (ANSI/IEEE[2], Navarrina *et al.*[4]).

The introduction of this hypothesis of circumferential uniformity allows to obtain approximated expressions of potential (3) and (5). Thus, being L the whole set of axial lines of the electrodes, $\widehat{\boldsymbol{\xi}}$ the orthogonal projection over the axis of a given generic point $\boldsymbol{\xi} \in \Gamma$, $\phi(\widehat{\boldsymbol{\xi}})$ the conductor diameter, and $\widehat{\sigma}(\widehat{\boldsymbol{\xi}})$ the approximated leakage current density at this point (assumed uniform around the cross section), expressions (3) and (5) result in

$$\widehat{V}_1(\boldsymbol{x}_1) = \frac{1}{4\gamma_1} \int_{\widehat{\boldsymbol{\xi}} \in L} \phi(\widehat{\boldsymbol{\xi}})\, \bar{k}_{11}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}})\, \widehat{\sigma}(\widehat{\boldsymbol{\xi}})\, dL, \quad \forall \boldsymbol{x}_1 \in E_1; \qquad (17)$$

$$\widehat{V}_2(\boldsymbol{x}_2) = \frac{1}{4\gamma_1} \int_{\widehat{\boldsymbol{\xi}} \in L} \phi(\widehat{\boldsymbol{\xi}})\, \bar{k}_{12}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}})\, \widehat{\sigma}(\widehat{\boldsymbol{\xi}})\, dL, \quad \forall \boldsymbol{x}_2 \in E_2. \qquad (18)$$

where $\bar{k}_{11}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}})$ and $\bar{k}_{12}(\boldsymbol{x}, \widehat{\boldsymbol{\xi}})$ are the average of kernels (4) and (6) around the cross section at $\widehat{\boldsymbol{\xi}}$ (Colominas[7], Aneiros[8]).

Now, since the leakage current is not exactly uniform around the cross section, boundary condition $V_1(\boldsymbol{\chi}) = 1$, $\boldsymbol{\chi} \in \Gamma$ will not be strictly satisfied at every point $\boldsymbol{\chi}$ on the electrode surface Γ, and variational form (7) will not verify anymore. However, if we restrict the class of trial functions to those with circumferential uniformity, for all members $\widehat{w}(\widehat{\boldsymbol{\chi}})$ of a suitable class of test functions defined on L, it must hold the weaker variational form

$$\int_{\widehat{\boldsymbol{\chi}} \in L} \phi(\widehat{\boldsymbol{\chi}})\, \widehat{w}(\widehat{\boldsymbol{\chi}}) \left(\frac{1}{4\gamma_1} \int_{\widehat{\boldsymbol{\xi}} \in L} \phi(\widehat{\boldsymbol{\xi}})\, \bar{\bar{k}}_{11}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})\, \widehat{\sigma}(\widehat{\boldsymbol{\xi}})\, dL - 1 \right) dL = 0, \quad (19)$$

where $\bar{\bar{k}}_{11}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})$ is the average of kernel $k_{11}(\widehat{\boldsymbol{\chi}}, \widehat{\boldsymbol{\xi}})$ in (4) around the cross sections at points $\widehat{\boldsymbol{\chi}}$ and $\widehat{\boldsymbol{\xi}}$ (Colominas[7], Aneiros[8]).

Resolution of variational statement (19) requires the discretization of the domain: the axial lines of electrodes of the grounding grid. Thus, for given sets of 1D boundary elements and trial functions defined on L, the whole set of the axial lines and the unknown approximated leakage current density $\widehat{\sigma}$ can be discretized. Furthermore, we can obtain a discretized version for approximated potential expressions (17) and (18). Finally, for a suitable selection of test functions defined on L, statement (19) is reduced to a system of linear equations similar to (13), although the computation of its coefficients implies integration on a 1D domain (Aneiros[8], Colominas *et al.*[9]).

The computing effort of this approximated 1D approach is drastically reduced in comparison with the 2D one, since the 1D discretization (the set of axial lines) will be much more simple. Furthermore, averaged kernels $\bar{k}_{11}(\widehat{x},\widehat{\boldsymbol{\xi}})$, $\bar{k}_{12}(\widehat{x},\widehat{\boldsymbol{\xi}})$ and $\bar{\bar{k}}_{11}(\widehat{\boldsymbol{\chi}},\widehat{\boldsymbol{\xi}})$ can be evaluated by using the suitable unexpensive approximations developed by Colominas[7], for the computation of average kernels involved in the grounding analysis in uniform soil models.

On the other hand, since computation of remaining line integrals is not obvious and standard quadratures cannot be used due to the ill-conditioning of integrands, we can performed suitable arrangements in the final expressions of the matrix coefficients (Aneiros[8]), so that we can use the highly efficient analytical integration techniques derived by Navarrina *et al.*[4] and Colominas *et al.*[7] for grounding systems in uniform soils.

5 Application to a Real Case

The Boundary Element formulation presented in this paper has been included in the computer aided design system "TOTBEM" developed by authors in recent years for the grounding analysis (Casteleiro *et al.*[11]). With this system, it has been possible to compute accurately earthing grids in uniform soil models of electrical substations of medium/big sizes, with acceptable computing requirements in memory storage and CPU time (Colominas *et al.*[10]).

In the resolution of real grounding systems embedded in two layered soils, the computing effort required can be very high, specially when conductivities of soil layers are very different ($|\kappa| \approx 1$), because the rate of convergence in the computation of average kernels $\bar{k}_{11}$, $\bar{k}_{12}$ and $\bar{\bar{k}}_{11}$ is very low, and it is necessary to evaluate a large number of terms of these series in order to obtain accurate results (Aneiros[8]).

In this paper, we present the analysis of the Balaidos II substation grounding (close to the city of Vigo in Spain), by using a uniform soil model and a two layer one. The earthing system (figure 1) is formed by a grid of 107 cylindrical conductors (diameter: 11.28 mm) buried to a depth of 80 cm, supplemented with 67 vertical rods (each one has a length of 2.5 m and a diameter of 14.0 mm). The Ground Potential Rise considered has been 10 kV. Characteristics of the soil models are presented in table I. The numerical model used in this analysis has been a Galerkin formulation, and the grid has been discretized in 241 linear elements.

Results, such as the equivalent resistance, total surge current and potential profiles on the earth surface along two different lines obtained with the BEM approach by using uniform and two layer soil models can be found in table I and figure 1. As it is shown, results noticeably vary when different soil models are used, and in consequence the grounding design parameters (equivalent resistance, the touch, step and mesh voltages, etc.) may significantly change. For this reason, it will be essential to perform the analysis of a system with this BEM technique although the computing cost increases, in cases where conductivity changes markedly with depth, in order to assure the safety of the installation.

It is important to notice that, in this case, the grounding analysis in the two layer soil model is very complicated because part of the grid is buried in the upper layer and other in the lower. Therefore, implementation of the numerical approach must be carefully performed, in order to consider the different arrangements of electrodes in the soil. Furthermore, since conductivites of layers are very different ($\kappa = -0.94$), the analysis requires an important computing effort.

Table I.—Balaidos II Substation: Results by using different soil models

Two Layer Soil Model		**Uniform Soil Model**	
Upper Layer Resistivity :	2000 Ω m	—	
Lower Layer Resistivity :	60 Ω m	—	
Height of Upper Layer :	1.2 m	Earth Resistivity :	60 Ω m
Fault Current :	14.7 kA	Fault Current :	24.9 kA
Equivalent Resistance :	0.681 Ω	Equivalent Resistance :	0.401 Ω
CPU Time (AXP 4000):	144.5 sec.	CPU Time (AXP 4000):	1.5 sec.

6 Conclusions

A numerical approach based on the BEM for the analysis of substation earthing systems embedded in layered soils has been presented. This formulation has been applied to the practical case of a grounded system in an equivalent two layer soil. Taking into account the real geometry of these systems, the general 2D approach can be rewritten in terms of an approximated 1D version. Moreover, since suitable arrangements can be done in the discretized expressions, it is possible to use the same analytical integration techniques developed by the authors for grounding analysis in uniform soils. Finally, the BEM formulation proposed is a general methodology that allows to obtain highly accurate results in the earthing analysis of electrical substations of medium/big sizes by using layered soil models.

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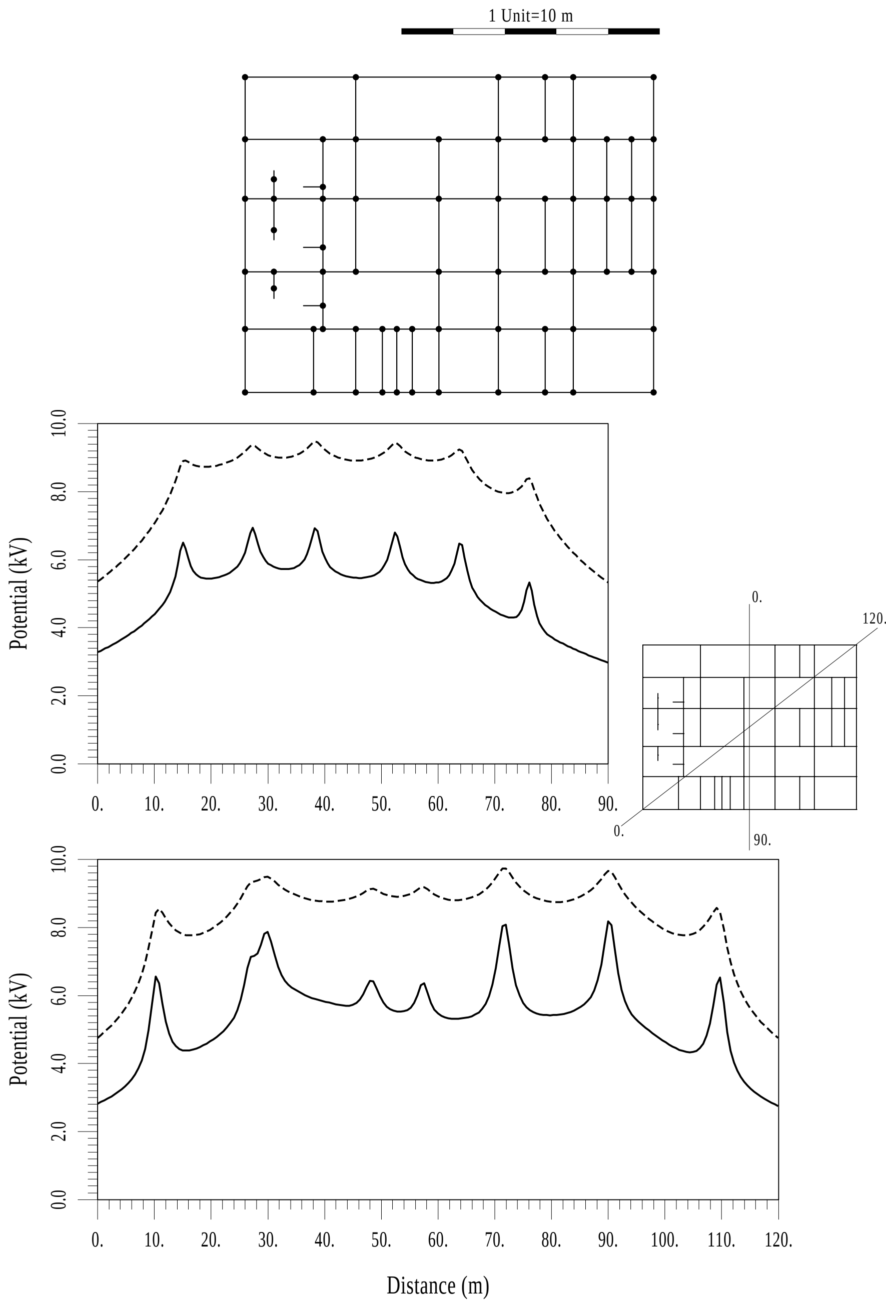

Fig. 1.— E.R. Balaídos II : Plan of the Grounding (vertical rods marked with black points), and Potential profiles along two different lines (in discontinuous line: results of the uniform model; in continuous line: results of the two layer model).