A NUMERICAL STUDY BASED ON THE FEM OF A MULTISCALE CONTINUUM MODEL FOR TUMOR ANGIOGENESIS

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Introduction

Cancer is a group of more than a hundred diseases with a central common characteristic: The uncontrolled division of abnormal cells. It has been present throughout human beings History and it is nowadays one of the leading death causes.

Cancer development embraces multiple scales (from subcellular to tissue scale) and it is a multi-step disease. One of the crucial phenomena that occurs during tumour growth is angiogenesis, i.e. the creation of new vessels from the pre-existing ones. The principal consequence of tumour angiogenesis is the destabilization of the avascular tumour, allowing its almost unrestrained growth, as well as providing means of spread through the entire organism.

Figure 1: Role of angiogenesis: New vessels allow the expansion of tumours. Experimental observations [Muthakkaruppan, 1982].

Judah Folkman hypothesized in 1971 [Folkman, 1971] the key role of angiogenesis in tumour growth and ever since many researches proposed mathematical models of angiogenesis. Due to the intricate morphology attained by the vessels, many authors have chosen discrete models [Chaplain, 2000, Stott, 1999], while few others have used a continuous description [Wise, 2008] or a mixture of them [Travasso, 2011].

Travasso et al developed a model which is founded on three equations: The continuous part of the model is defined by a reaction-diffusion equation for the growth factor evolution and a phase-field equation which describes endothelial cells, and the discrete part of the model is defined by the third equation which governs the velocity associated to the tip endothelial cells, treated here as discrete agents.

Methods

Here we propose a finite element method using isogeometric analysis for the approximation of the tumour angiogenesis theory proposed by the aforementioned authors. We performed a parametric study of the theory and compared the numerical results with relevant clinical data.

A secondary study has been made where we have analysed the best way to include the tip endothelial cells into the continuous formulation, varying the free energy functional used in the phase-field equation. We have also considered the inclusion of some other biological phenomena (e.g. the natural growth factor decay) and the possibility of coupling the system with the tumour evolution.

Results and discussion

We have made several numerical simulations of the previous model in order to understand the theory they proposed. We synthesised the results in a phase diagram.

References

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